

Cap. 11

11.26



DET A VELOCIDADE DE ESCAPE

$$v = 0 \rightarrow r \rightarrow \infty$$

$$a = -g \frac{R^2}{r^2}$$

$$a ds = v dv$$

$$\int -g \frac{R^2}{r^2} dr = \int v dv$$

$$-gR^2 \int_R^\infty r^{-2} dr = \int_v^0 v dv$$

$$-gR^2 \left(-\frac{1}{r} \right) \Big|_R^\infty = \frac{v^2}{2} \Big|_v^0$$

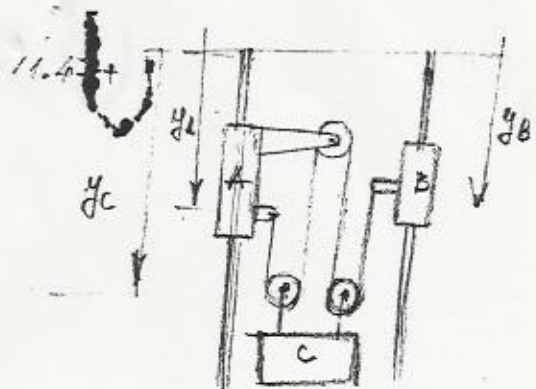
$$gR^2 \left(\frac{1}{\infty} - \frac{1}{R} \right) = 0 - \frac{v^2}{2}$$

$$gR^2 \left(\frac{1}{\infty} - \frac{1}{R} \right) = -\frac{v^2}{2}$$

$$v^2 = 2gR$$

$$v = \sqrt{2gR}$$





$$3(y_C - y_A) + y_C - y_B = L$$

$$3y_C - 3y_A + y_C - y_B = L$$

$$4y_C - 3y_A - y_B = L$$

$$\frac{d}{dt} 4v_C - 3v_A - v_B = 0$$

$$\frac{d}{dt} 4a_C - 3a_A - a_B = 0$$

$$v_{0A} = 0$$

$$a_A = 6,25 \text{ cm/s}^2 \uparrow$$

$$v_{0B} = 37,5 \text{ cm/s} \downarrow$$

$$a_B = 0$$

$$4v_C - 3v_A - v_B = 0$$

$$4v_C - 0 - (-37,5) = 0$$

$$v_C = -9,375 \text{ m/s} \downarrow$$

$$4a_C - 3 \times 6,25 - 0 = 0$$

$$a_C = 4,69 \text{ m/s}^2 \uparrow$$

$$v_C = v_{0C} + at$$

$$0 = -9,37 + 4,69t$$

$$t = 2 \text{ s}$$

$$\Delta y = v_{0C}t + \frac{1}{2}at^2$$

$$\Delta y = -9,37 \times 2 + \frac{1}{2} \times 4,69 \times 2^2$$

$$\Delta y = 9,38 \text{ cm} \downarrow$$

11.48 -

$$v_{0A} = 0$$

$$v_{0B} = 0$$

$$a_A = 7,5t \text{ cm/s}^2 \uparrow$$

$$a_B = 22,5 \text{ cm/s}^2 \downarrow$$

$$4a_C - 3a_A - a_B = 0$$

$$4a_C - 3 \times 7,5t - (-22,5) = 0$$

$$4a_C - 22,5t + 22,5 = 0$$

$$a_C = 5,62t - 5,62$$

$$a_C = \frac{dv_C}{dt}$$

$$\int_0^v dv_C = \int_0^t (5,62t - 5,62) dt$$

$$v = \frac{5,62t^2}{2} - 5,62t$$

$$v = 0 \Rightarrow t \left(\frac{5,62t}{2} - 5,62 \right) = 0$$

$$t = 0$$

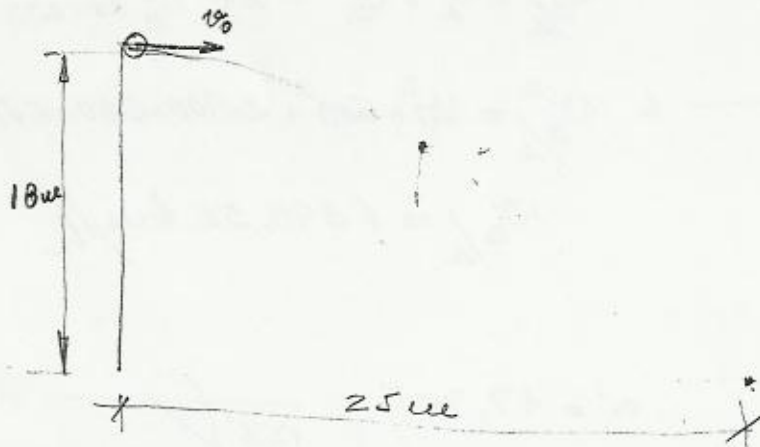
$$t = 2 \text{ s}$$

$$\int_0^y dy = \int_0^2 \left(\frac{5,62t^2}{2} - 5,62t \right) dt$$

$$\Delta y = \left[\frac{5,62t^3}{6} - \frac{5,62t^2}{2} \right]_0^2$$

$$\Delta y = 3,75 \text{ cm} \downarrow$$

11.87-



$$v_{0x} = v_0$$

$$v_{0y} = 0$$

$$y = \frac{1}{2} g t^2$$

$$18 = \frac{1}{2} \times 9,81 \times t^2$$

$$t = 1,92 \text{ s.}$$

$$x = v_0 t$$

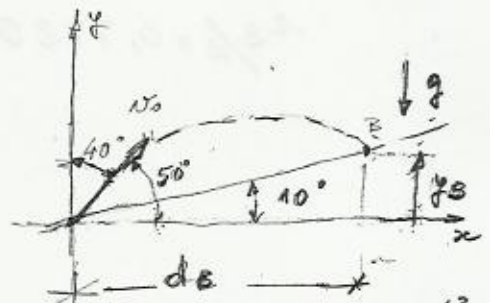
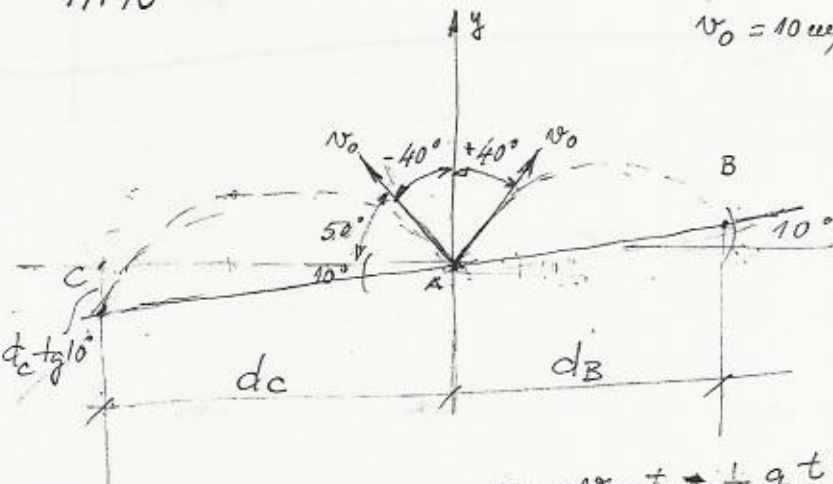
$$25 = v_0 \times 1,92$$

$$v_0 = 13,05 \text{ m/s}$$

11.93



$$v_0 = 10 \text{ m/s}$$



$$10 \cos 50^\circ \sin 10^\circ t = 10 \cos 40^\circ t - \frac{1}{2} 9,81 t^2$$

$$-6,527 t = -\frac{1}{2} 9,81 t^2$$

$$t \left(\frac{1}{2} 9,81 t - 6,527 \right) = 0$$

$$t = 0$$

$$t = 1,33 \text{ s}$$

$$d_B = 10 \cos 50^\circ \times t$$

$$d_B = 8,55 \text{ m}$$

$$y_C = d_C \tan 10^\circ$$

$$d_C = v_{0x} t$$

$$d_C = 10 \cos 50^\circ \times t$$

$$y_C = v_{0y} t + \frac{1}{2} g t^2$$

$$-d_C \tan 10^\circ = 10 \cos 40^\circ t + \frac{1}{2} 9,81 t^2$$

$$-10 \cos 50^\circ \tan 10^\circ t = 10 \cos 40^\circ t + \frac{1}{2} 9,81 t^2$$

$$-8,79 t = -\frac{1}{2} 9,81 t^2$$

$$t \left(\frac{1}{2} 9,81 t - 8,79 \right) = 0$$

$$t = 0$$

$$t = 1,79 \text{ s}$$

$$d_C = 10 \cos 50^\circ \times t$$

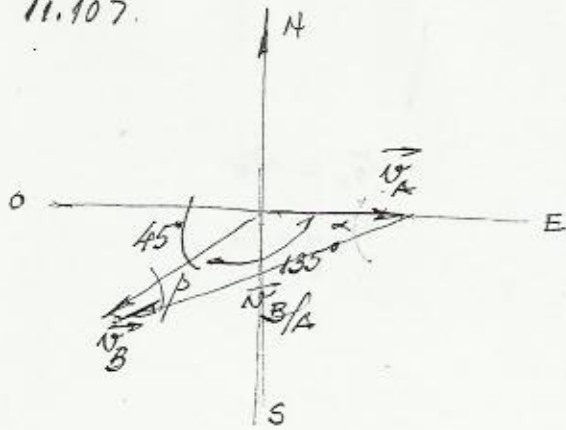
$$d_C = 11,50 \text{ m}$$

$$y_B = v_{0y} t - \frac{1}{2} g t^2$$

$$y_B = d_B \tan 10^\circ$$

$$d_B = v_{0x} t$$

11.107.



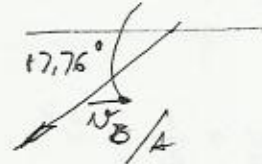
$$v_{B/A}^2 = v_A^2 + v_B^2 - 2v_A v_B \cos 135^\circ$$

$$v_{B/A}^2 = 900 + 600^2 - 2 \times 900 \times 600 \times \cos 135^\circ$$

$$v_{B/A} = 1.390,56 \text{ km/h}$$

$$\frac{v_B}{\sin \alpha} = \frac{v_{B/A}}{\sin 135^\circ}$$

$$\alpha = 17,76^\circ$$



$$t = 2 \text{ s}$$

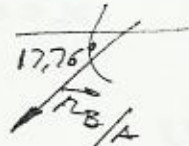
$$r_{B/A} = v_{B/A} t$$

$$t = \frac{2}{3600} \text{ h}$$

$$r_{B/A} = 1.390,56 \times \frac{2}{3600}$$

$$r_{B/A} = 0,7725 \text{ km}$$

$$r_{B/A} = 772,54 \text{ m}$$

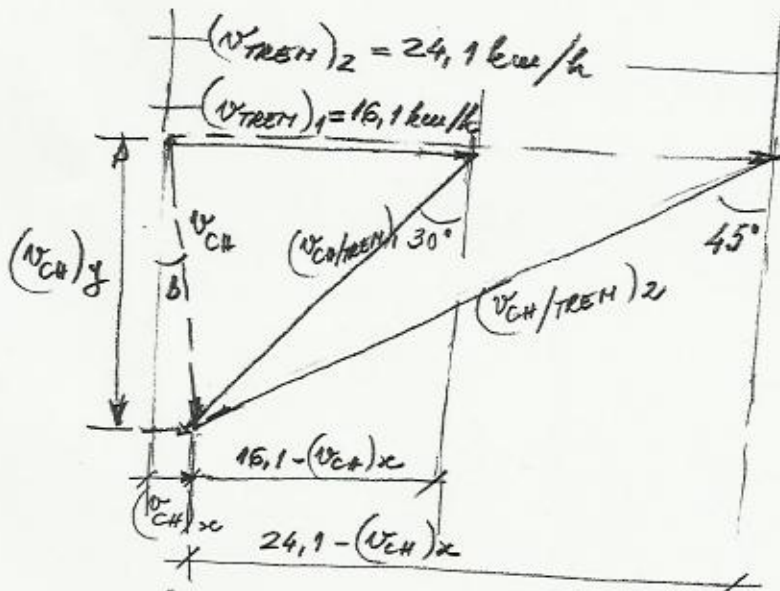


11.117

$$v_{CH} = v_{TREM} + v_{CH/TREM}$$

CASO ① : $v_{TREM} = 16,1 \text{ km/h}$; $v_{CH/TREM} \nearrow 30^\circ$

CASO ② : $v_{TREM} = 24,1 \text{ km/h}$; $v_{CH/TREM} \nearrow 45^\circ$



$$\tan 30^\circ = \frac{(v_{CH})_y}{16,1 - (v_{CH})_x}$$

$$\tan 45^\circ = \frac{(v_{CH})_y}{24,1 - (v_{CH})_x}$$

$$(v_{CH})_y = \tan 30^\circ [16,1 - (v_{CH})_x]$$

$$(v_{CH})_y = \tan 45^\circ [24,1 - (v_{CH})_x]$$

$$\tan 30^\circ [16,1 - (v_{CH})_x] = \tan 45^\circ [24,1 - (v_{CH})_x]$$

$$0,5774 (16,1 - (v_{CH})_x) = 24,1 - (v_{CH})_x$$

$$0,42 (v_{CH})_x = 14,80$$

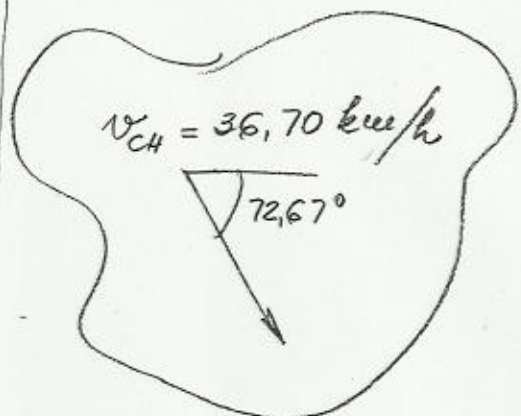
$$(v_{CH})_x = 35,03 \text{ km/h}$$

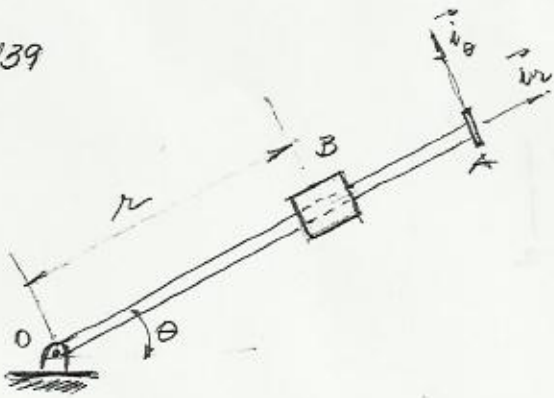
$$(v_{CH})_y = -10,93 \text{ km/h}$$

$$v_{CH} = \sqrt{(35,03)^2 + (-10,93)^2}$$

$$v_{CH} = 36,70 \text{ km/h}$$

$$\tan \beta = \frac{35,03}{10,93} \quad \beta = 72,67^\circ$$





$$\theta = t^3 - 4t$$

$$r = 25t^3 - 50t^2$$

$$\dot{\theta} = \frac{d\theta}{dt} = 3t^2 - 4 \quad \dot{r} = 75t^2 - 100t$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = 6t \quad \ddot{r} = 150t - 100$$

$$t = 1s$$

$$a) \quad \vec{v} = \dot{r} \vec{i}_r + r \dot{\theta} \vec{i}_\theta$$

$$\vec{v} = (75t^2 - 100t) \vec{i}_r + (25t^3 - 50t^2)(3t^2 - 4) \vec{i}_\theta$$

$$\vec{v} = (75t^2 - 100t) \vec{i}_r + (75t^5 - 150t^4 - 100t^3 + 200t^2) \vec{i}_\theta$$

$$b) \quad \vec{a} = (\ddot{r} - r \dot{\theta}^2) \vec{i}_r + (r \ddot{\theta} + 2 \dot{r} \dot{\theta}) \vec{i}_\theta$$

$$\vec{v} = -25 \text{ mm/s } \vec{i}_r + 25 \text{ mm/s } \vec{i}_\theta$$

$$\vec{a} = [(150t - 100) - (25t^3 - 50t^2)(3t^2 - 4)^2] \vec{i}_r + [(25t^3 - 50t^2)6t + 2(75t^2 - 100t)3t]$$

$$t = 1s$$

$$\vec{a} = [50 - (-25)(-1)] \vec{i}_r + [-150 + (-25)(-1)]$$

$$\vec{a} = 25 \text{ mm/s}^2 \vec{i}_r - 125 \text{ mm/s}^2 \vec{i}_\theta$$

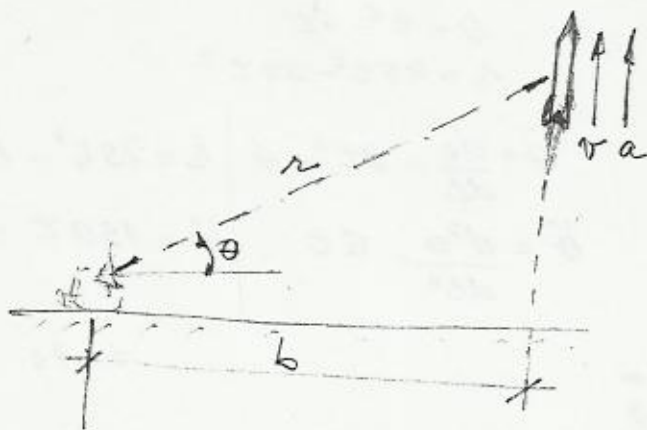
c) A aceleração do cursor em relação a haste é \ddot{r} :

$$\vec{a}_{B/OA} = \ddot{r} \vec{i}_r$$

$$\vec{a}_{B/OA} = 150t - 100$$

$$P / t = 1s$$

$$\vec{a}_{B/OA} = 50 \text{ mm/s}^2$$



$$r = \frac{b}{\cos\theta} = b \cos^{-1}\theta = b \sec\theta$$

$$\vec{r} = r_r \vec{i}_r + r_\theta \vec{i}_\theta$$

$$\vec{v} = \dot{r} \vec{i}_r + r \dot{\theta} \vec{i}_\theta$$

$$\dot{r} = -b \cos^{-2}\theta (-\sin\theta) \dot{\theta}$$

$$v_r = \dot{r} = b \sec\theta \tan\theta \dot{\theta}$$

$$v_\theta = b \sec\theta \dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2}$$

$$v = \sqrt{b^2 \sec^2 \theta \tan^2 \theta \dot{\theta}^2 + b^2 \sec^2 \theta \dot{\theta}^2}$$

$$v = b \sec\theta \dot{\theta} \sqrt{1 + \tan^2 \theta}$$

$$v = b \sec^2 \theta \dot{\theta}$$

$$v = \frac{b}{\cos^2 \theta} \dot{\theta}$$

11.149 -

$$\theta = 42.0^\circ \rightarrow \theta = 43.2^\circ$$

$$\Delta\theta = 1.2^\circ$$

$$\Delta t = 0.5 \text{ s}$$

$$\dot{\theta} \approx \frac{\Delta\theta}{\Delta t} = \frac{1.2^\circ}{0.5 \text{ s}}$$

$$\dot{\theta} = 2.4^\circ/\text{s} = 0.0419 \text{ rad/s}$$

$$b = 3 \text{ km} = 3.000 \text{ m}$$

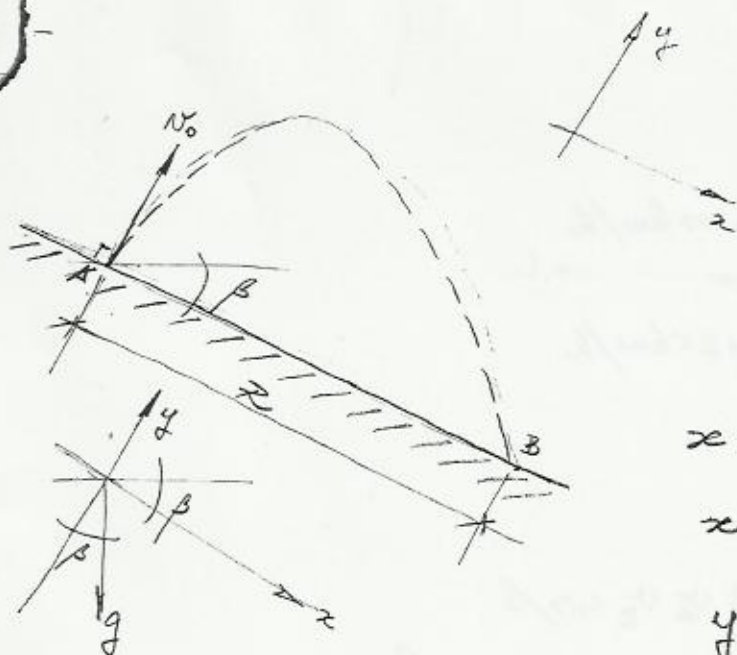
$$\theta = 43.2^\circ$$

$$v = \frac{3000 \times 0.0419}{(\cos 43.2^\circ)^2}$$

$$v = 236.54 \text{ m/s}$$

$$v = 851.54 \text{ km/h}$$

EXERCÍCIOS: 10; 25; 26; 30; 31; 35; 40; 41; 42; 47; 48; 53; 56; 88; 90; 92; 93;
94; 97; 99; 100; 108; 113; 114; 120; 127; 132; 135; 139; 140; 145; 146; 147; 148; 155; 156



$$a_x = g \sin \beta \quad v_{0x} = 0$$

$$a_y = -g \cos \beta \quad v_{0y} = v_0$$

$$x = 0 + \frac{1}{2} a_x t^2$$

$$x = \frac{1}{2} g \sin \beta t^2$$

$$y = v_0 t - \frac{1}{2} g \cos \beta t^2$$

$$\text{P/ } x = R$$

$$y = 0$$

$$0 = v_0 t - \frac{1}{2} g \cos \beta t^2$$

$$t \left(\frac{1}{2} g \cos \beta t - v_0 \right) = 0$$

$$t = 0 \text{ e } t = \frac{2v_0}{g \cos \beta}$$

$$x = \frac{1}{2} g \sin \beta t^2$$

$$R = \frac{1}{2} g \sin \beta \left(\frac{2v_0}{g \cos \beta} \right)^2$$

$$R = \frac{1}{2} g \sin \beta \frac{4v_0^2}{g^2 \cos^2 \beta}$$

$$R = \frac{2v_0^2}{g} \operatorname{tg} \beta \sec \beta$$

$$11.170 - v_0 = 10 \text{ m/s}$$

$$\beta = 30^\circ$$

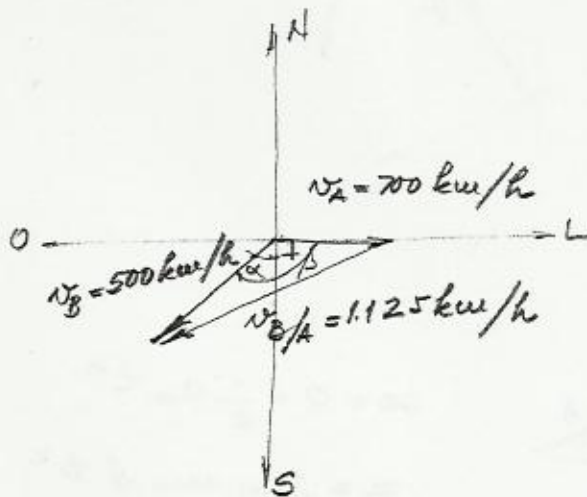
$$g = 9,81 \text{ m/s}^2$$

$$R = \frac{2 \cdot v_0^2}{g} \operatorname{tg} \beta \cdot \sec \beta$$

$$R = \frac{2 \times 10^2}{9,81} \times \operatorname{tg} 30^\circ \frac{1}{\cos 30^\circ}$$

$$R = 13,59 \text{ m}$$

11.171 -



$$v_{B/A}^2 = v_A^2 + v_B^2 - 2 v_A v_B \cos \beta$$
$$1.125^2 = 700^2 + 500^2 - 2 \times 700 \times 500 \cos \beta$$

$$\cos \beta = -0.7509$$

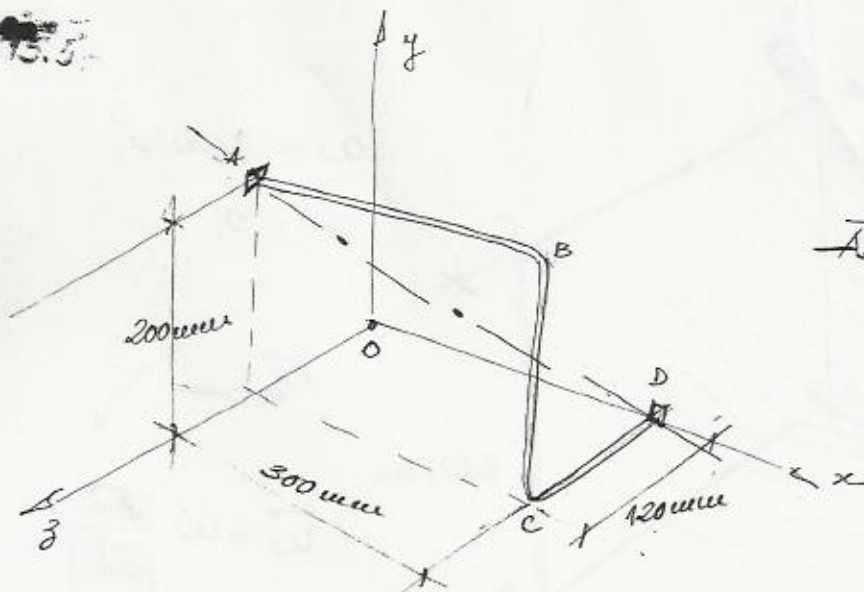
$$\beta = 138.67^\circ$$

$$\beta = \alpha + 90^\circ$$

$$\Rightarrow \boxed{\alpha = 48.67^\circ}$$

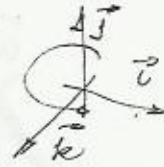


Cap. 15



$$\omega = 95 \text{ rad/s}$$

$$\vec{AB} = 0,3 \text{ m } \vec{c}$$



$$\vec{\omega} = \omega \frac{\vec{AD}}{|\vec{AD}|} = 95 \frac{0,3\vec{c} - (0,2\vec{j} + 0,12\vec{k})}{\sqrt{0,3^2 + 0,2^2 + 0,12^2}}$$

$$\vec{\omega} = 75 \text{ rad/s } \vec{c} - 50 \text{ rad/s } \vec{j} - 30 \text{ rad/s } \vec{k}$$

$$\vec{v}_B = \vec{\omega} \wedge \vec{AB}$$

$$\vec{v}_B = (75\vec{c} - 50\vec{j} - 30\vec{k}) \wedge 0,3\vec{c} = +50 \times 0,3 \vec{k} - 30 \times 0,3 \vec{j}$$

$$\vec{v}_B = (-9,0 \text{ m/s}) \vec{j} + (15 \text{ m/s}) \vec{k}$$

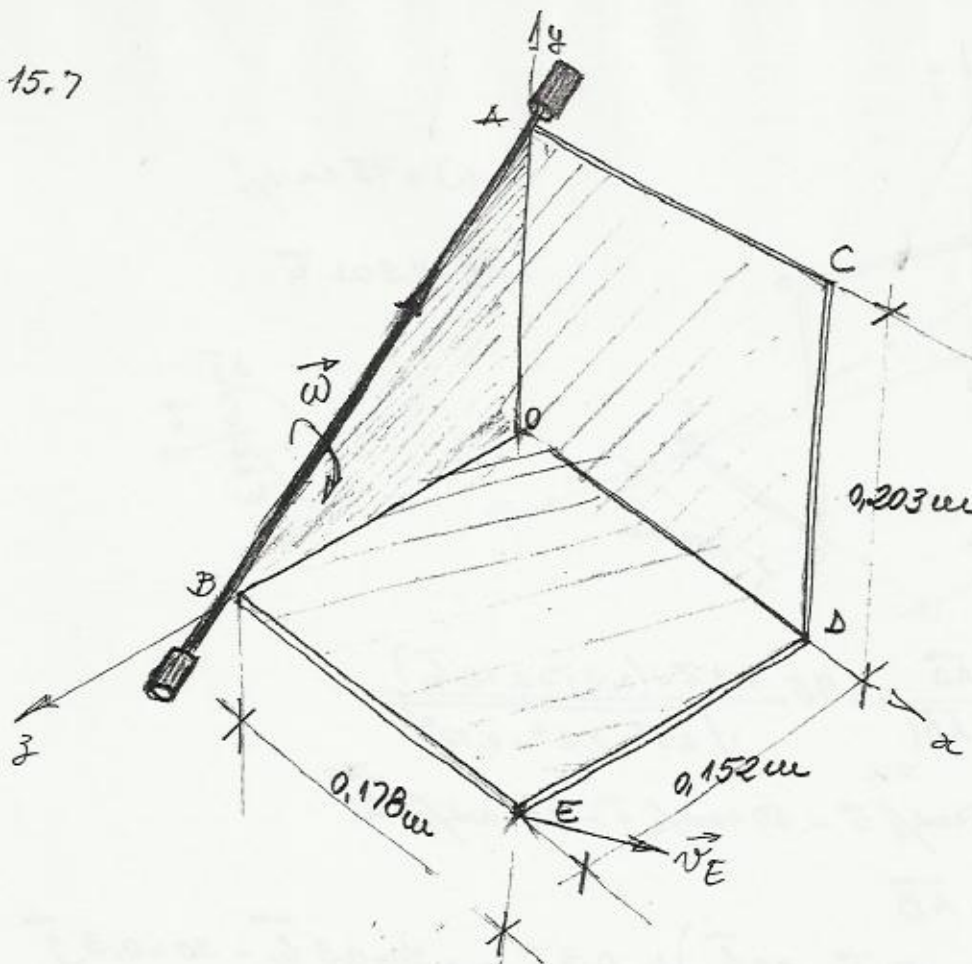
$$\vec{a}_B = \vec{\alpha} \wedge \vec{AB} + \vec{\omega} \wedge (\vec{\omega} \wedge \vec{AB}) \quad \vec{\alpha} = 0$$

$$\vec{a}_B = (75\vec{c} - 50\vec{j} - 30\vec{k}) \wedge (-9\vec{j} + 15\vec{k})$$

$$\vec{a}_B = -675\vec{k} - 1.125\vec{j} - 750\vec{c} - 270\vec{c}$$

$$\vec{a}_B = (-1020 \text{ m/s}^2) \vec{c} + (-1.125 \text{ m/s}^2) \vec{j} + (-675 \text{ m/s}^2) \vec{k}$$

15.7

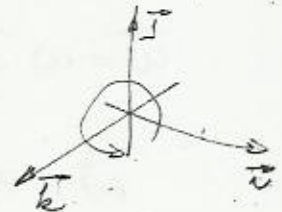


$$\omega = 5 \text{ rad/s}$$

$$\alpha = 0$$

$$\vec{v}_E = \vec{\omega} \times \vec{r}_{E/O}$$

$$\vec{\omega} = \omega \frac{\vec{BA}}{|\vec{BA}|}$$



$$\vec{\omega} = \frac{5 + 0.203\vec{j} + 0.152\vec{k}}{\sqrt{0.203^2 + 0.152^2}}$$

$$\vec{\omega} = (-4 \text{ rad/s})\vec{j} + (-3 \text{ rad/s})\vec{k}$$

$$\vec{v}_D = \vec{\omega} \wedge \vec{BD} = (4\vec{j} - 3\vec{k}) \wedge (0.178\vec{i} - 0.152\vec{k})$$

$$\vec{v}_D = -0.71\vec{k} - 0.61\vec{i} + 0.53\vec{j}$$

$$\boxed{\vec{v}_D = (-0.6 \text{ m/s})\vec{i} + (0.53 \text{ m/s})\vec{j} + (-0.71 \text{ m/s})\vec{k}}$$

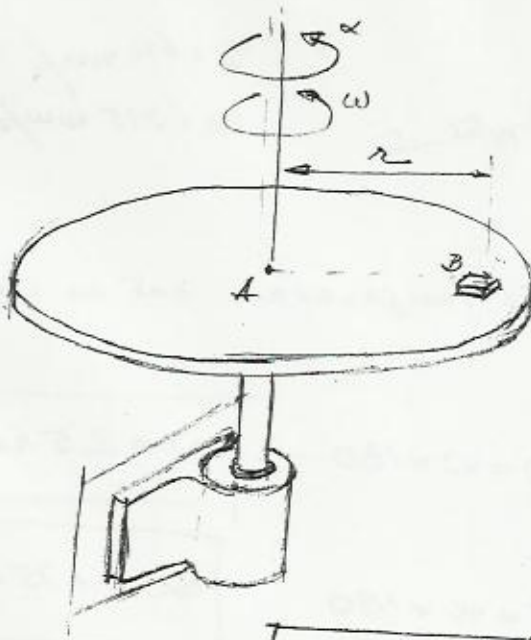
$$\vec{a}_D = \vec{\alpha} \wedge \vec{BD} + \vec{\omega} \wedge (\vec{\omega} \wedge \vec{BD}) \quad \vec{\alpha} = 0$$

$$\vec{a}_D = (4\vec{j} - 3\vec{k}) \wedge (-0.6\vec{i} - 0.53\vec{j} - 0.71\vec{k})$$

$$\vec{a}_D = +2.4\vec{k} - 2.84\vec{i} + 1.8\vec{j} - 1.59\vec{i}$$

$$\boxed{\vec{a}_D = (-4.43 \text{ m/s}^2)\vec{i} + (1.8 \text{ m/s}^2)\vec{j} + (2.4 \text{ m/s}^2)\vec{k}}$$

15.15



$$a = 3 \text{ m/s}^2$$

$$t = 0 \rightarrow \omega_0 = 0$$

$$\alpha = 4 \text{ rad/s}^2$$

$$r = 200 \text{ mm}$$

$$a_t = \alpha r$$

$$a_n = \omega^2 r$$

$$\omega = \omega_0 + \alpha t$$

$$\omega = \alpha t$$

$$a_n = \alpha^2 r t^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$a = \sqrt{\alpha^2 r^2 + (\alpha^2 r t^2)^2}$$

$$a = \alpha r \sqrt{1 + \alpha^2 t^4}$$

$$3 = 4 \times 0,2 \sqrt{1 + 4^2 t^4}$$

$$3 = 0,8 \sqrt{1 + 16 t^4}$$

$$3,75 = \sqrt{1 + 16 t^4}$$

$$(3,75)^2 = 1 + 16 t^4$$

$$t = 0,95 \text{ s}$$

15.16

$$\alpha = 0,5 \text{ rad/s}^2 \quad r = 0,2 \text{ m}$$

$$a = \alpha r \sqrt{1 + \alpha^2 t^4}$$

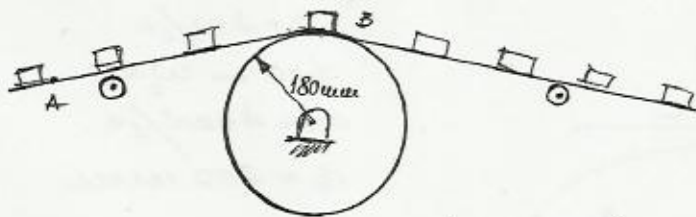
$$a = 0,1 \sqrt{1 + 0,25 t^4}$$

a) $t = 0 \quad a_0 = 0,10 \text{ m/s}^2$

b) $t = 1 \text{ s} \quad a_1 = 0,11 \text{ m/s}^2$

c) $t = 2 \text{ s} \quad a_2 = 0,22 \text{ m/s}^2$

15.13-



$$v_A = 450 \text{ mm/s} \leftarrow$$

$$a_A = 315 \text{ mm/s}^2 \rightarrow$$

As velocidades e aceleração tangenciais são as mesmas para toda a correia.

a)

$$v_B = v_A = 450 \text{ mm/s}$$

$$v_B = \omega r \quad 450 = \omega \times 180$$

$$\omega = 2,5 \text{ rad/s} \curvearrowright$$

$$a_{Bt} = a_A = 315 \text{ mm/s}^2$$

$$a_{Bt} = \alpha r \quad 315 = \alpha \times 180$$

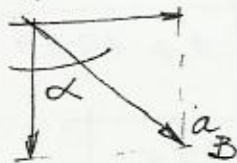
$$\alpha = 1,75 \text{ rad/s}^2 \curvearrowright$$

b)

$$\vec{a}_B = a_{Bt} \vec{u}_t + a_{Bn} \vec{u}_n$$

$$a_{Bt} = 315 \text{ mm/s}^2 \rightarrow$$

$$a_{Bn} = \omega^2 r = 2,5^2 \times 180 = 1125 \text{ mm/s}^2 \downarrow$$



$$\tan \alpha = \frac{315}{1125}$$

$$a_B = \sqrt{315^2 + 1125^2}$$

$$a_B = 1.168,27 \text{ mm/s}^2$$

$$a_B = 1,17 \text{ m/s}^2$$

15,64°

15.14

$$\omega = 3 \text{ rad/s} \curvearrowright$$

$$a_B = 2 \text{ m/s}^2$$

$$r = 0,18 \text{ m}$$

$$a_B^2 = a_{Bt}^2 + a_{Bn}^2$$

$$a_{Bt} = \alpha r = 0,18\alpha$$

$$a_{Bn} = \omega^2 r = 3^2 \times 0,18 = 1,62 \text{ m/s}^2$$

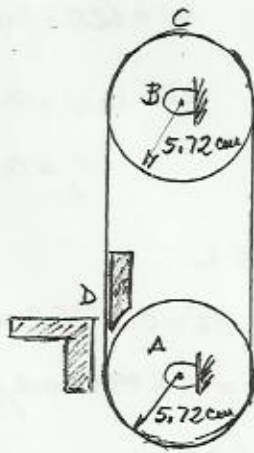
$$a_B = 2 \text{ m/s}^2$$

$$2^2 = (0,18\alpha)^2 + 1,62^2$$

$$\alpha = \pm 6,52 \text{ rad/s}^2$$

(sentido horário ou anti-horário)

15.17-



$\omega_0 = 0$

$\alpha = 120 \text{ rad/s}^2$

Let. a_c .

a_D .

$t = 2 \text{ s}$

$\omega = \omega_0 + \alpha t$

$\omega = 0 + 120 \times 2$

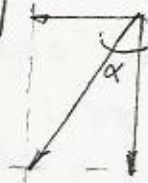
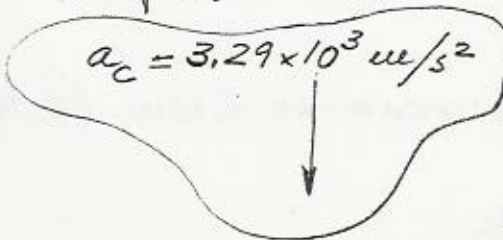
$\omega = 240 \text{ rad/s}$

$a_{ct} = \alpha r = 120 \times 5.72 = 686.40 \text{ cm/s}^2 = 6.86 \text{ m/s}^2$

$a_{cm} = \omega^2 r = 240^2 \times 5.72 = 329.472 \text{ cm/s}^2 = 3.294.72 \text{ m/s}^2$

$a_c = \sqrt{a_{ct}^2 + a_{cm}^2}$

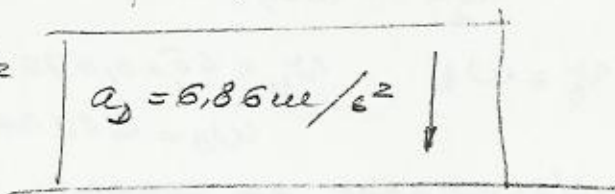
$a_c = \sqrt{6.86^2 + 3.294.72^2}$



$\tan \alpha = \frac{6.86}{3.294.72}$

$\alpha = 0.12^\circ$

$a_D = a_{ct} = 6.86 \text{ m/s}^2$

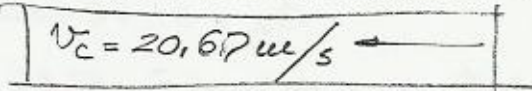


15.18-

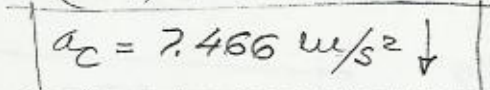
$\omega = 3450 \text{ rpm}$ $1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s} = \frac{\pi}{30} \text{ rad/s}$

$\omega = 115\pi \text{ rad/s}$

a) $v_c = \omega r = 115\pi \times 5.72 \text{ cm} = 2066.54 \text{ cm/s}$



$a_c = \omega^2 r = (115\pi)^2 \times 5.72 = 746.605.96 \text{ cm/s}^2$

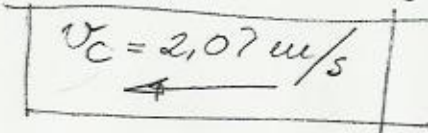


b) CÁLCULO DE α : $\omega = \omega_0 + \alpha t$ $0 = 115\pi - \alpha \times 5$ $\alpha = 72.26 \text{ rad/s}^2$

$t = 4.5 \text{ s}$ $\omega = 115\pi - 72.26 \times 4.5$

$\omega = 36.13 \text{ rad/s}$

$v_c = 36.13 \times 0.0572$



$a_m = \omega^2 r$ $a_m = 74.67 \text{ m/s}^2$

$a_t = \alpha r$ $a_t = 4.13 \text{ m/s}^2$

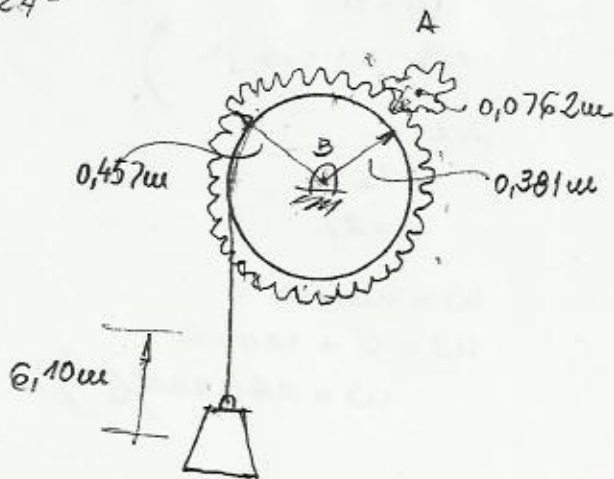
$a = \sqrt{74.67^2 + 4.13^2}$

$a_c = 74.78 \text{ m/s}^2$



$\theta = 86.83$

15.24-



$\omega_0 = 0$ roda A

$\omega = 120 \text{ rpm} = 120 \times \frac{\pi}{30} = 4\pi \text{ rad/s}$

$\omega = \omega_0 + \alpha t \quad t = 5 \text{ s}$

$\alpha_A = 0,8\pi \text{ rad/s}^2$

$a_t = \alpha r$

$a_t = 0,8\pi \times 0,0762 = \alpha_B \times 0,457$

$\alpha_B = 0,4191 \text{ rad/s}^2$

ACELERAÇÃO NA CORDA.

$a = 0,4191 \times 0,381 \quad a = 0,16 \text{ m/s}^2$

DESLOCAMENTO DO PESO EM 5 s.

$\Delta s = \frac{1}{2} a t^2 \quad \Delta s = \frac{1}{2} \times 0,16 \times 5^2$

$\Delta s = 2 \text{ m}$

RESTA AO PESO DESLOCAR-SE 4,10m (6,10m - 2m)

A PARTIR DE 5 s:

$\omega_A = 4\pi \text{ rad/s} = \text{cte.}$

$v_t = \omega r \quad v_t = 4\pi \times 0,0762 = \omega_B \times 0,457$

$\omega_B = 2,10 \text{ rad/s}$

VELOCIDADE DA CORDA:

$v = \omega_B \times 0,381 = 0,798 \text{ m/s}$

$\Delta s = vt \quad 4,1 = 0,798 t \quad t = 5,14 \text{ s}$

a) CÁLCULO DO Nº DE ROTACIONES DA ENGRENAGEM A

0 - 5 s. $\theta_A = \frac{1}{2} \alpha_A t^2 \quad \theta_A = \frac{1}{2} \times 0,8\pi \times 5^2$

$\theta_A = 31,42 \text{ rad}$

5 s - 10,14 s. $\theta_A = \omega_A t$

$\theta_A = 4\pi \times 5,14$

$\theta_A = 64,59 \text{ rad}$

$\theta_A = 31,42 + 64,59$

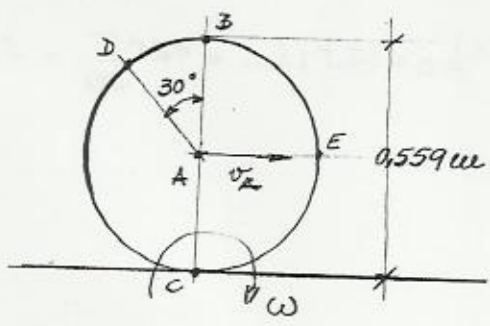
$\theta_A = 96,01 \text{ rad}$

$\theta_A = 15,3 \text{ rot.}$

b) $t = 5 + 5,14$

$t = 10,14 \text{ s}$

15.30-



$v_A = 72,4 \text{ km/h} \rightarrow$

$\omega = \frac{72,4}{3,6 \times \frac{0,559}{2}}$

$\omega = 71,95 \text{ rad/s}$

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = 20,11 \text{ m/s} + \frac{71,95 \times 0,559}{2}$

$v_B = 40,22 \text{ m/s}$

$\vec{v}_C = \vec{v}_A + \vec{v}_{C/A} = 20,11 \text{ m/s} + 20,11 \text{ m/s}$

$v_C = 0$

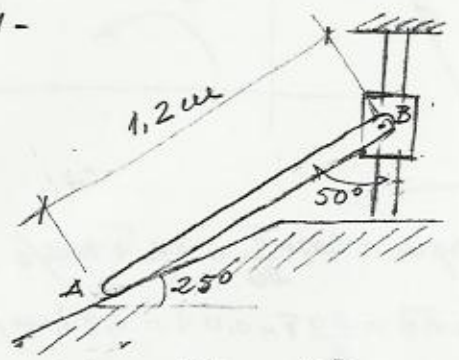
$\vec{v}_D = \vec{v}_A + \vec{v}_{D/A} = 20,11 \text{ m/s} + \frac{71,95 \times 0,559}{2}$
 $\swarrow 30^\circ$

$v_D = 38,85 \text{ m/s}$
 $\swarrow 15^\circ$

$\vec{v}_E = \vec{v}_A + \vec{v}_{E/A} = 20,11 \text{ m/s} + \frac{71,95 \times 0,559}{2}$
 \downarrow

$v_E = 28,44 \text{ m/s}$
 $\swarrow 45^\circ$

15.31-



$v_B = 1,8 \text{ m/s} \uparrow$

$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$

$v_A = 1,8 + \omega \times 1,2$
 $\swarrow 25^\circ$ \uparrow $\swarrow 50^\circ$

Comp. \rightarrow :

$v_A \cos 25^\circ = \omega \times 1,2 \cos 50^\circ$

$v_A = 0,85\omega$

Comp. \uparrow :

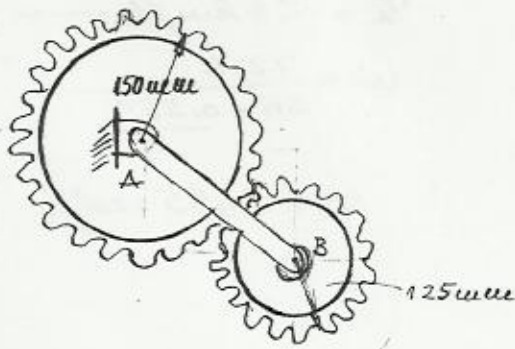
$v_A \sin 25^\circ = 1,8 - \omega \times 1,2 \sin 50^\circ$

$0,85\omega \sin 25^\circ + \omega \times 1,2 \sin 50^\circ = 1,8$

$\omega = 1,41 \text{ rad/s}$

$v_A = 0,85\omega$

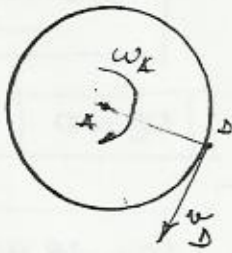
$v_A = 1,20 \text{ m/s}$
 $\swarrow 25^\circ$



$$\omega_A = 150 \text{ rpm} = 150 \times \frac{\pi}{30} = 5\pi \text{ rad/s}$$

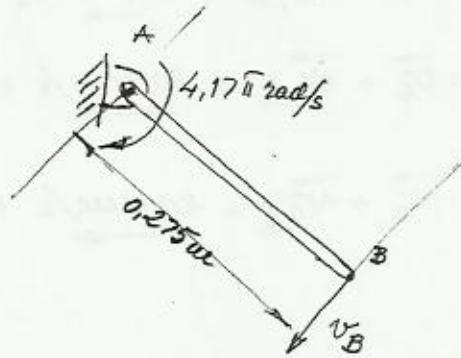
$$\omega_{AB} = 125 \text{ rpm} = 125 \frac{\pi}{30} = 4,17\pi \text{ rad/s}$$

RODA A:



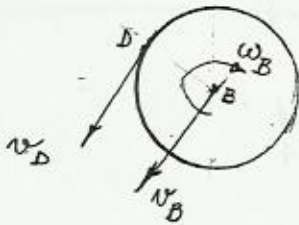
$$v_D = \omega_A \times 0,15 = 2,36 \text{ m/s}$$

BALOK AB



$$v_B = 4,17\pi \times 0,275 = 3,60 \text{ m/s}$$

RODA B.



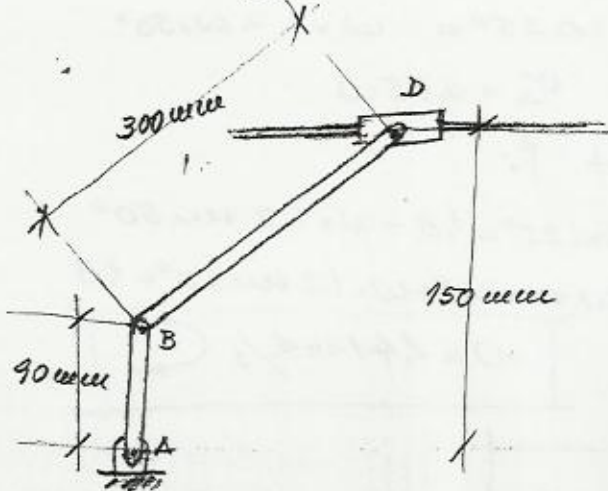
$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

$$2,36 = 3,6 - \omega_B \cdot r_B$$

$$\omega_B = 9,95 \text{ rad/s}$$

15.44 -

$\theta = 0^\circ$



$$\omega_{AB} = 200 \text{ rpm} = 200 \times \frac{\pi}{30} = \frac{20}{3} \pi \text{ rad/s}$$

$$v_B = \omega_{AB} \times AB = \frac{20}{3} \pi \times 0,09 = 1,89 \text{ m/s}$$

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

$$v_D = 1,89 + \omega_{BD} \times BD$$

COMP. \uparrow :

$$0 = \omega_{BD} \times 0,3 \sin \theta$$

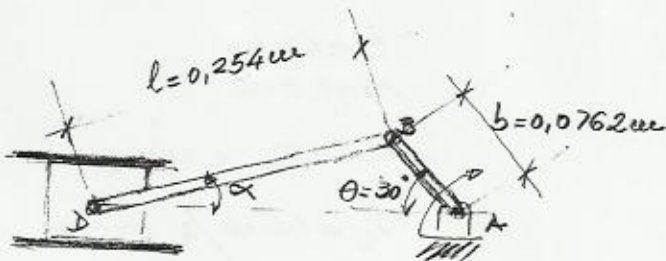
COMP. \rightarrow

$$v_D = 1,89$$

$$\omega_{BD} = 0$$

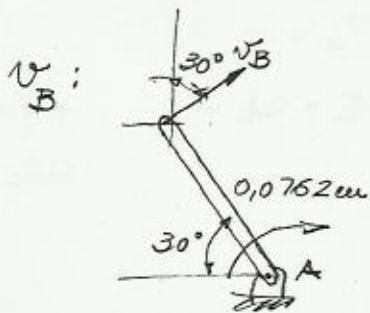
$$v_D = 1,89 \text{ m/s}$$

15.46-



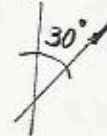
$$\omega_{AB} = 750 \text{ rpm}$$

$$\omega_{AB} = 25\sqrt{2} \text{ rad/s}$$



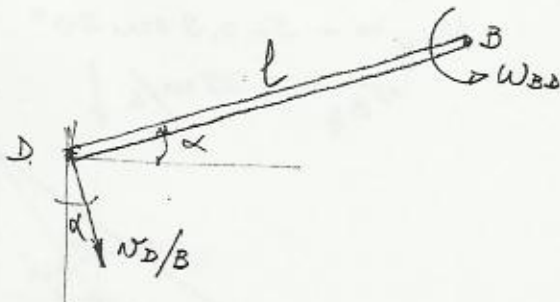
$$v_B = 25\sqrt{2} \times 0,0762$$

$$v_B = 5,98 \text{ m/s}$$



$$\frac{0,254}{\sin 30^\circ} = \frac{0,0762}{\sin \alpha}$$

$$\alpha = 8,63^\circ$$



$$v_{D/B} = \omega_{BD} \times l$$

$$v_{D/B} = 0,254 \omega_{BD}$$



$$v_D = v_{D/B}$$

$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

$$v_D = 5,98 + 0,254 \omega_{BD}$$

Comp. \uparrow :

$$0 = 5,98 \cos 30^\circ - 0,254 \omega_{BD} \cos 8,63^\circ$$

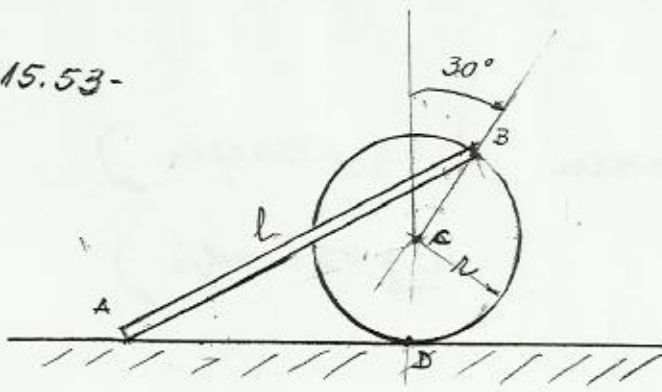
$$\omega_{BD} = 20,64 \text{ rad/s}$$

Comp. \rightarrow :

$$v_D = 5,98 \sin 30^\circ + 0,254 \omega_{BD} \sin 8,63^\circ$$

$$v_D = 3,78 \text{ m/s}$$

15.53-



$$l = 1,2 \text{ m}$$

$$r = 0,3 \text{ m}$$

Det. v_A e ω_{AB}

$$v_C = 1,5 \text{ m/s} \rightarrow$$

$$v_D = 0$$

$$v_C = \omega_C \times r \quad 1,5 = \omega_C \times 0,3$$

$$\omega_C = 5 \text{ rad/s}$$

$$\vec{v}_B = \vec{v}_C + \vec{v}_{B/C}$$

$$v_B = 1,5 + \omega_C \times 0,3$$



Comp \rightarrow :

$$v_{Bx} = 1,5 + 5 \times 0,3 \cos 30^\circ$$

$$v_{Bx} = 2,80 \text{ m/s} \rightarrow$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$v_A = v_B + \omega_{AB} \times \overline{AB}$$

Comp \uparrow :

$$0 = -0,75 + 1,2 \omega_{AB} \cos 27,81^\circ$$

$$\omega_{AB} = 0,71 \text{ rad/s}$$

Comp \rightarrow :

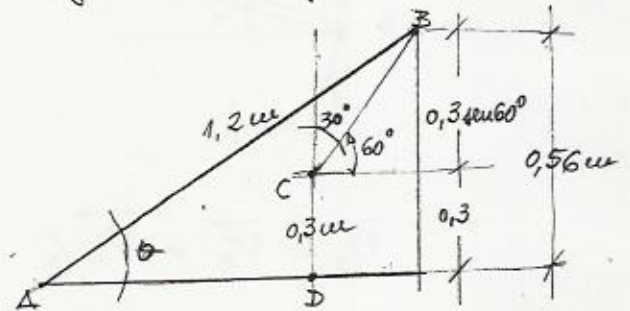
$$v_A = 2,8 - 1,2 \times 0,71 \sin 27,81^\circ$$

$$v_A = 2,40 \text{ m/s}$$

Comp \uparrow :

$$-v_{By} = -5 \times 0,3 \sin 30^\circ$$

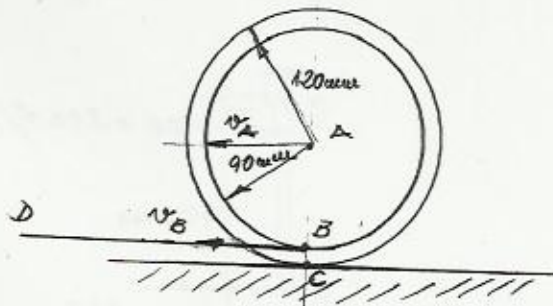
$$v_{By} = 0,75 \text{ m/s} \downarrow$$



$$\sin \theta = \frac{0,56}{1,2}$$

$$\theta = 27,81^\circ$$

15.57-



$$v_B = v_B = 150 \text{ mm/s}$$

$$v_C = 0$$

C - C.I.

$$v_B = (120 - 90) \times \omega$$

$$150 = 30 \times \omega$$

$$a) \quad \boxed{\omega = 5 \text{ rad/s}} \quad \curvearrowright$$

$$b) \quad v_A = \omega r$$

$$v_A = \omega \times 120$$

$$v_A = 5 \times 120$$

$$\boxed{v_A = 600 \text{ mm/s} \leftarrow}$$

$$c) \quad v_A > v_B$$

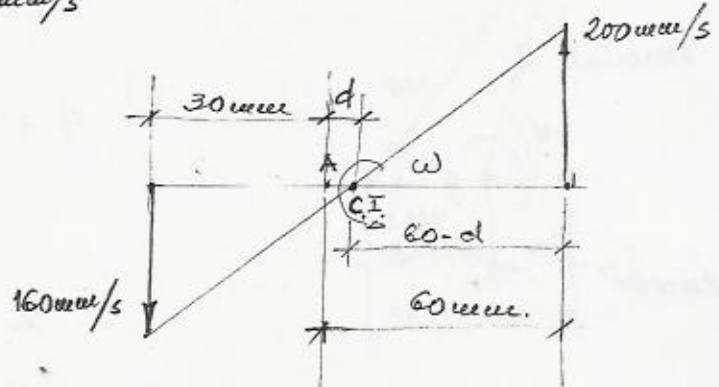
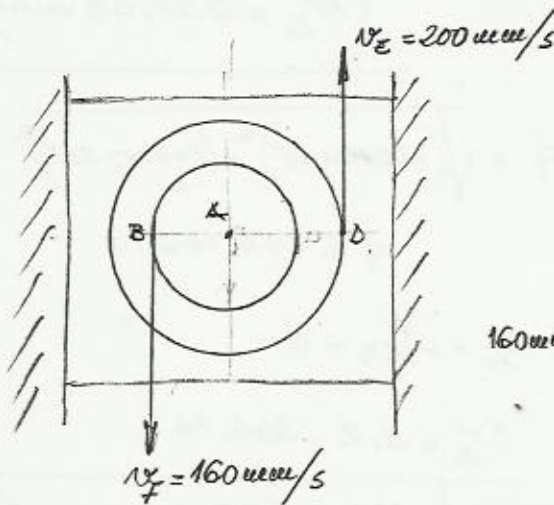
$$\Delta s = v_A / v_B \cdot t$$

$$\Delta s = (v_A - v_B) \times t$$

$$\Delta s = (600 - 150) \times 1$$

$$\boxed{\Delta s = 450 \text{ mm.}}$$

15.58-



$$\frac{160}{30+d} = \frac{200}{60-d} \quad \frac{60-d}{30+d} = \frac{200}{160}$$

$$a) \quad 60 - d = (30 + d) \cdot 1,25$$

$$60 - d = 37,5 + 1,25d$$

$$60 - 37,5 = 2,25d$$

$$\boxed{d = 10 \text{ mm}}$$

$$b) \quad v_{\text{seco}} = v_A$$

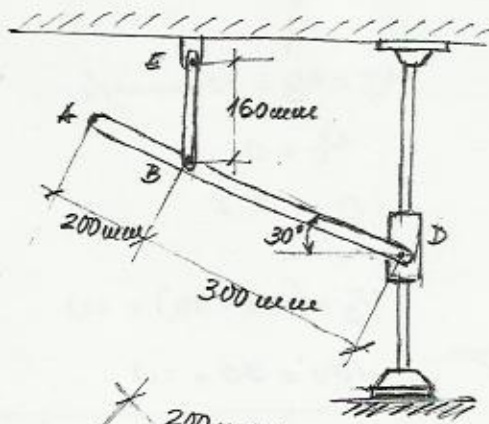
$$v_A = \omega d \quad \omega = \frac{160}{40} = 4 \text{ rad/s}$$

$$v_A = 4 \times 10$$

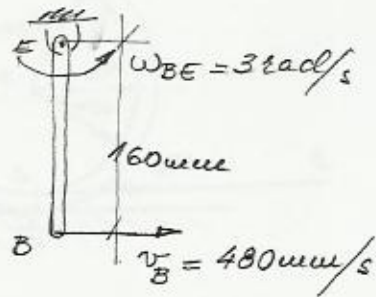
$$\boxed{v_A = 40 \text{ mm/s} \downarrow}$$

c) POLIA EXTERNA -
 $v_{E/A} = 240 \text{ mm/s} \uparrow$
 VEENROCK 240 mm/s

POLIA INTERNA.
 $v_{F/A} = 120 \text{ mm/s} \downarrow$
 VEENROCK 120 mm/s



$$\omega_{BE} = 3 \text{ rad/s}$$



$$v_B = \omega_{AD} \times 300 \sin 30^\circ$$

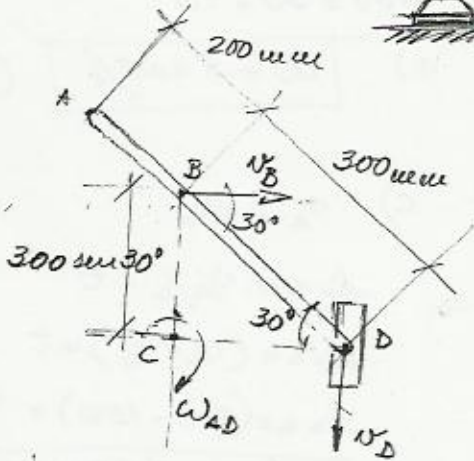
$$a) \frac{480}{300 \sin 30^\circ} = \omega_{AD}$$

$$\omega_{AD} = 3,2 \text{ rad/s}$$

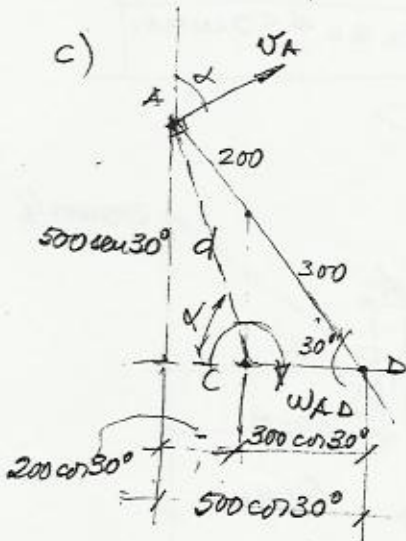
$$b) v_D = \omega_{AD} \times 300 \cos 30^\circ$$

$$v_D = 3,2 \times 300 \cos 30^\circ$$

$$v_D = 831,38 \text{ mm/s}$$



$$c) \tan \alpha = \frac{500 \sin 30^\circ}{200 \cos 30^\circ}$$



$$c) d = \sqrt{(500 \sin 30^\circ)^2 + (200 \cos 30^\circ)^2}$$

$$d = 304,14 \text{ mm}$$

$$v_A = \omega_{AD} \times d$$

$$v_A = 3,2 \times 304,14$$

$$v_A = 973,24 \text{ mm/s}$$

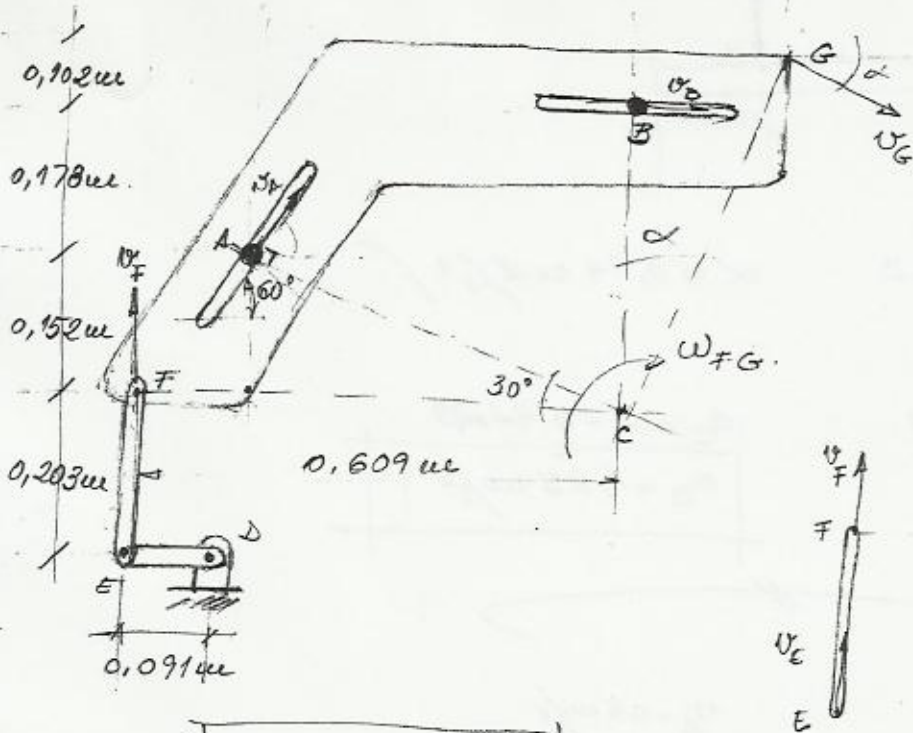


$$\tan \alpha = \frac{500 \sin 30^\circ}{200 \cos 30^\circ}$$

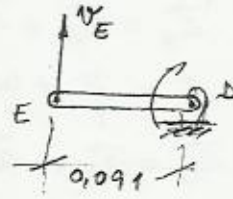
$$\tan \alpha = 1,44$$

$$\alpha = 55,29^\circ$$

15.67- $0,152\text{ m}$ $0,457\text{ m}$ $0,203\text{ m}$



$\omega_{DE} = 8 \text{ rad/s}$



$v_E = 0,728 \text{ m/s}$

$C \pm \rightarrow \infty$

$\omega_{FE} = 0$

$v_F = v_E = 0,728 \text{ m/s}$

a) $v_F = 0,728 \text{ m/s}$

b) $v_F = \omega_{FG} \times 0,609$

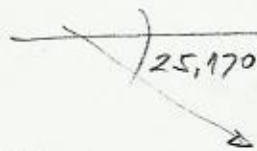
$0,728 = \omega_{FG} \times 0,609 \quad \omega_{FG} = 1,195 \text{ rad/s}$

$\overline{CG} = \sqrt{(0,152 + 0,178 + 0,102)^2 + 0,203^2}$

$\overline{CG} = 0,477 \text{ m}$

$v_G = \omega_{FG} \times \overline{CG}$

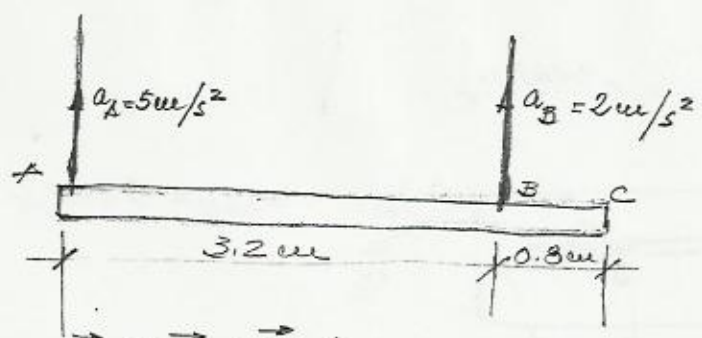
$v_G = 0,57 \text{ m/s}$



$\text{tg} \alpha = \frac{0,203}{0,432}$

$\alpha = 25,170^\circ$

15.79-



$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

$$\uparrow = \uparrow + \uparrow \quad \alpha = 0,94 \text{ rad/s}^2 \curvearrowright$$

$$5 = 2 + \alpha \times 3,2$$

$$\vec{a}_C = \vec{a}_B + \vec{a}_{C/B}$$

$$\uparrow = \uparrow + \downarrow$$

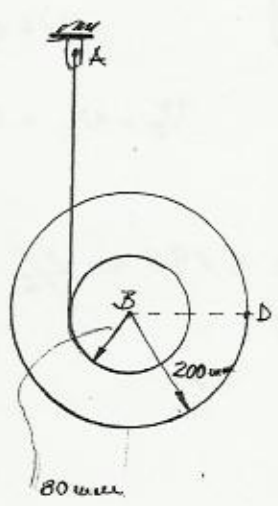
$$\vec{a}_C = 2 + \alpha \times 0,8$$

$$\alpha = 0,94 \text{ rad/s}^2 \curvearrowright$$

$$a_C = 2 - 0,94 \times 0,8$$

$a_C = 1,25 \text{ m/s}^2 \uparrow$

15.84-



$$v_B = 0,6 \text{ m/s}$$

$$a_B = 2,4 \text{ m/s}^2$$

$$v_B = \omega r$$

$$0,6 = \omega \times 0,08 \quad \omega = 7,5 \text{ rad/s} \curvearrowright$$

$$a_B = \alpha r$$

$$2,4 = \alpha \times 0,08 \quad \alpha = 30 \text{ rad/s}^2 \curvearrowright$$

$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$$

$$\vec{a}_D = a_B + \alpha \times 0,2 + \omega^2 \times 0,2$$

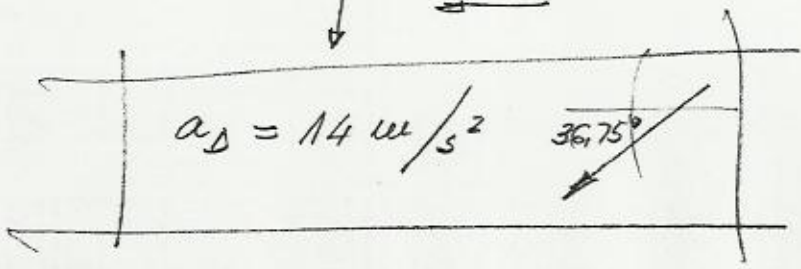
$$\downarrow \quad \downarrow \quad \leftarrow$$

$$\vec{a}_D = (2,4 + 30 \times 0,2) + 7,5^2 \times 0,2$$

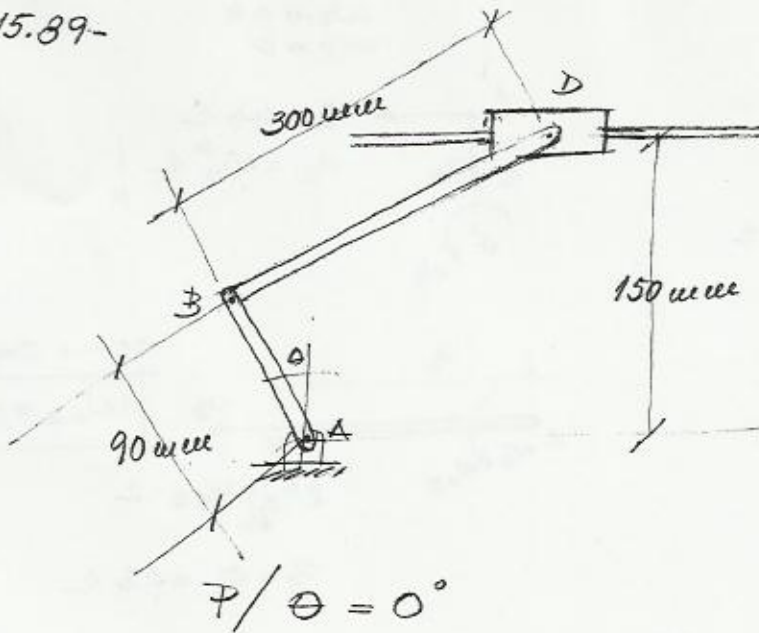
$$\downarrow \quad \leftarrow$$

$$\vec{a}_D = 8,4 + 11,25$$

$$\tan \alpha = \frac{8,4}{11,25} \quad \alpha = 36,75^\circ$$



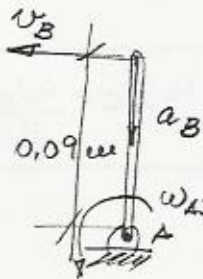
15.89-



$$\omega_{AB} = 0$$

$$\omega_{AB} = 200 \text{ rpm}$$

$$\omega_{AB} = \frac{200 \times \pi}{30} = 20,94 \text{ rad/s}$$



$$v_B = \omega r = 20,94 \times 0,09 = 1,88 \text{ m/s}$$

$$a_B = \omega^2 r = 20,94^2 \times 0,09 = 39,46 \text{ m/s}^2$$

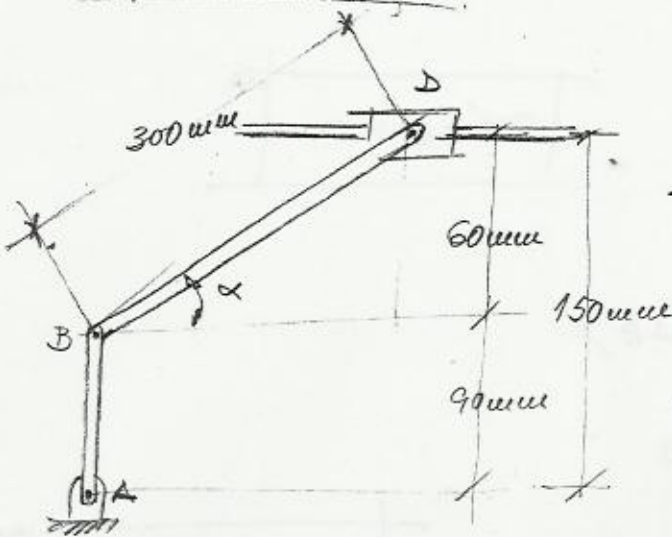
$$\vec{v}_D = \vec{v}_B + \vec{v}_{D/B}$$

$$v_D = v_B + \omega_{BD} \cdot BD$$

Comp \uparrow :

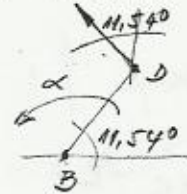
$$0 = \omega_{BD} \cdot BD \sin \alpha$$

$$\omega_{BD} = 0$$



$$\vec{a}_D = \vec{a}_B + \vec{a}_{D/B}$$

$$a_D = 39,46 + \alpha \cdot 0,3$$



Comp \uparrow :

$$0 = -39,46 + \alpha \times 0,3 \cos 11,54^\circ$$

$$\alpha = 134,26 \text{ rad/s}$$

Comp \rightarrow :

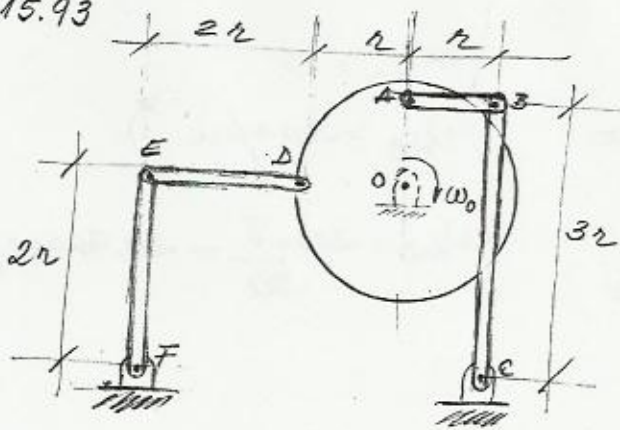
$$-a_D = -\alpha \times 0,3 \sin 11,54^\circ$$

$$a_D = 8,06 \text{ m/s}^2$$

$$\sin \alpha = \frac{60}{300}$$

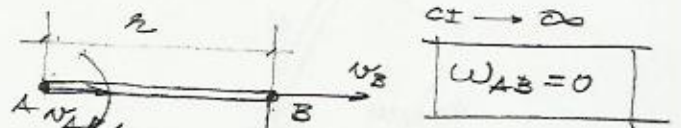
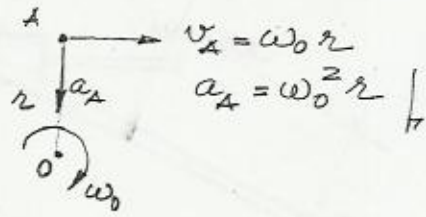
$$\alpha = 11,54^\circ$$

15.93



$$\omega_0 = \text{cte.}$$

$$\alpha_0 = 0$$



$\text{ct} \rightarrow \infty$

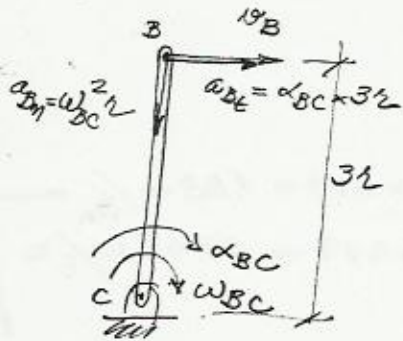
$$\omega_{AB} = 0$$

$$v_B = v_A = \omega_0 r$$

$$v_B = \omega_{BC} \times 3r$$

$$\omega_0 r = \omega_{BC} \times 3r$$

$$\omega_{BC} = \frac{\omega_0}{3}$$



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\omega_{BC} \times 3r \quad \omega_0^2 r \quad \omega_{AB} \cdot r$$

Comp \rightarrow :

$$\omega_{BC} \times 3r = 0$$

$$\omega_{BC} = 0$$

Comp \uparrow :

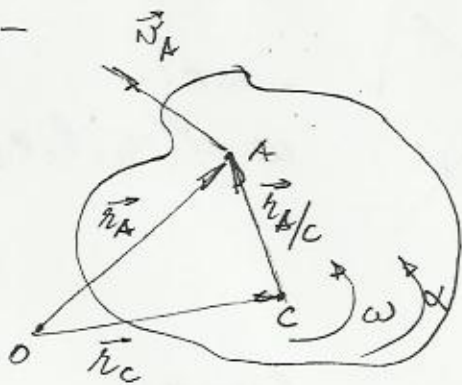
$$-\omega_{BC}^2 \times 3r = -\omega_0^2 r - \omega_{AB} r$$

$$\frac{3\omega_0^2}{9} - \omega_0^2 = \omega_{AB}$$

$$\frac{\omega_0^2}{3} - \omega_0^2 = \omega_{AB}$$

$$\omega_{AB} = -\frac{2}{3} \omega_0^2$$

$$\omega_{AB} = -\frac{2}{3} \omega_0^2$$



C — CENTRO INSTANTANEO DE ROTACÃO

$$\vec{v}_C = 0$$

$$\vec{r}_{A/C} = \vec{r}_A - \vec{r}_C$$

$$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C}$$

$$\vec{v}_A = \vec{\omega} \wedge \vec{r}_{A/C} \quad (\text{MULTIPLICANDO VETORIALMENTE POR } \vec{\omega})$$

$$\vec{\omega} \wedge \vec{v}_A = \vec{\omega} \wedge \vec{\omega} \wedge \vec{r}_{A/C}$$

$$\vec{\omega} = \omega \vec{k}$$

$$\vec{\omega} \perp \vec{r}_{C/A}$$

$$\vec{\omega} \wedge \vec{v}_A = -\omega^2 \vec{r}_{A/C}$$

$$\vec{\omega} \wedge \vec{v}_A = -\omega^2 (\vec{r}_A - \vec{r}_C)$$

$$\vec{r}_{A/C} = -\frac{\vec{\omega} \wedge \vec{v}_A}{\omega^2}$$

$$\vec{r}_C = \vec{r}_A + \frac{\vec{\omega} \wedge \vec{v}_A}{\omega^2}$$

$$\vec{a}_A = \vec{a}_C + \vec{a}_{A/C}$$

$$\vec{a}_C = 0$$

$$\vec{\omega} = \omega \vec{k}$$

$$\vec{\alpha} = \alpha \vec{k}$$

$$\vec{\alpha} \perp \vec{r}_{C/A}$$

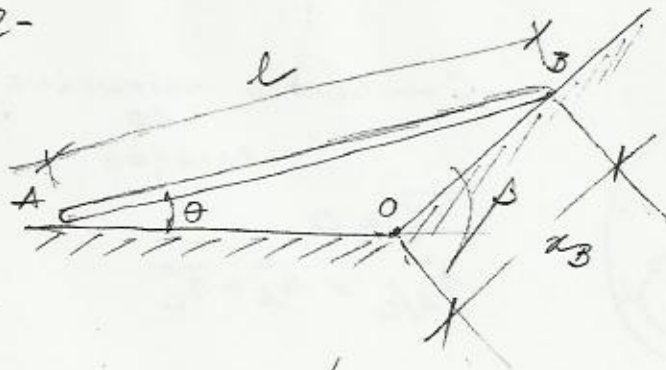
$$\vec{a}_A = \vec{\alpha} \wedge \vec{r}_{A/C} + \vec{\omega} \wedge \vec{\omega} \wedge \vec{r}_{A/C}$$

$$\vec{a}_A = \vec{\alpha} \wedge \left(\frac{-\vec{\omega} \wedge \vec{v}_A}{\omega^2} \right) + \vec{\omega} \wedge \vec{\omega} \wedge \left(\frac{-\vec{\omega} \wedge \vec{v}_A}{\omega^2} \right)$$

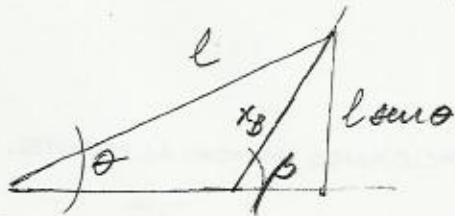
$$\vec{a}_A = \frac{\alpha \omega \vec{v}_A}{\omega^2} + (-\omega^2) \left(\frac{-\vec{\omega} \wedge \vec{v}_A}{\omega^2} \right)$$

$$\vec{a}_A = \frac{\alpha}{\omega} \vec{v}_A + \vec{\omega} \wedge \vec{v}_A$$

15.102-



DET:
 $\omega = f(x_B, l, \theta, \beta)$



$$\text{sen } \beta = \frac{l \text{ sen } \theta}{x_B}$$

$$x_B = \frac{l \text{ sen } \theta}{\text{sen } \beta}$$

$$v_B = \frac{d x_B}{dt}$$

$$v_B = \frac{l \text{ cos } \theta \dot{\theta}}{\text{sen } \beta}$$

$$\dot{\theta} = \omega$$

$$v_B = \frac{l \text{ cos } \theta \omega}{\text{sen } \beta}$$

$$\omega = \frac{v_B \text{ sen } \beta}{l \text{ cos } \theta}$$

15.103

DET. $\alpha = f(v_B, l, \theta, \beta)$

$$a_B = 0$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \frac{\text{sen } \beta}{l} v_B^{-1} \text{ cos } \theta$$

$$\frac{d\omega}{dt} = \frac{\text{sen } \beta}{l} \left(\frac{d v_B^{-1} \text{ cos } \theta}{dt} \right) + \frac{\text{sen } \beta}{l} \left(v_B^{-2} \cdot (-\text{cos } \theta) \cdot (-\text{sen } \theta) \dot{\theta} \right)$$

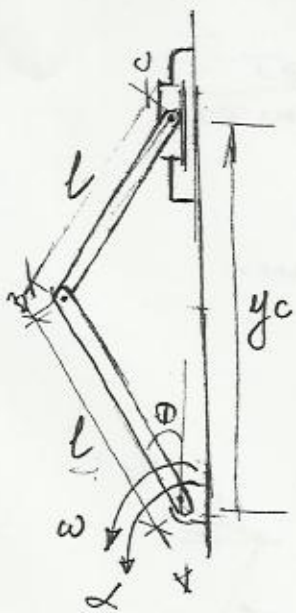
$$a_B = \frac{d v_B}{dt} = 0$$

$$\dot{\theta} = \omega = \frac{v_B \text{ sen } \beta}{l \text{ cos } \theta}$$

$$\alpha = \frac{\text{sen } \beta}{l} \cdot \frac{v_B \text{ sen } \theta}{\text{cos}^2 \theta} \dot{\theta}$$

$$\alpha = \left(\frac{v_B \text{ sen } \beta}{l} \right)^2 \frac{\text{sen } \theta}{\text{cos}^3 \theta}$$

15.107-



$$\omega = \dot{\theta}$$

$$\alpha = \ddot{\theta}$$

$$y_c = 2l \cos \theta$$

$$v_c = \frac{dy_c}{dt} = -2l \sin \theta \dot{\theta}$$

$$v_c = -2l \sin \theta \omega$$

$$v_c = 2l \omega \sin \theta$$

$$a_c = \frac{dv_c}{dt}$$

$$a_c = \frac{d}{dt} (-2l \sin \theta \dot{\theta})$$

$$a_c = -2l \cos \theta \dot{\theta} \dot{\theta} + (-2l \sin \theta \ddot{\theta})$$

$$a_c = 2l (\alpha \sin \theta + \omega^2 \cos \theta)$$

15.108 - $v_c = v_0$
 $a_c = 0$

DET. ω e α

$$v_c = 2l \omega \sin \theta \quad v_0 = 2l \omega \sin \theta$$

$$\omega = \frac{v_0}{2l \sin \theta}$$

$$a_c = -2l (\sin \theta \alpha + \omega^2 \cos \theta)$$

$$a_c = 0$$

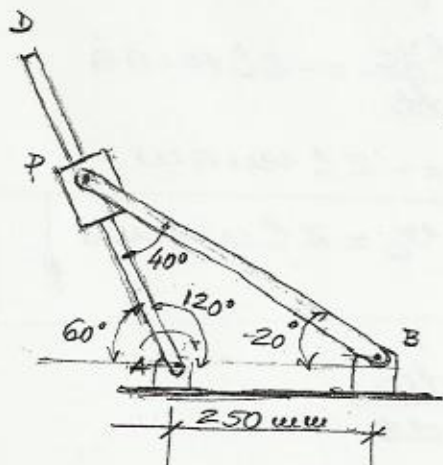
$$\sin \theta \alpha = -\omega^2 \cos \theta$$

$$\alpha = -\left(\frac{v_0}{2l \sin \theta}\right)^2 \frac{\cos \theta}{\sin \theta}$$

$$\alpha = -\left(\frac{v_0}{2l}\right)^2 \frac{\cos \theta}{\sin^3 \theta}$$

$$\alpha = \left(\frac{v_0}{2l}\right)^2 \frac{\cos \theta}{\sin^3 \theta}$$

$$\omega_A = 8 \text{ rad/s} \quad \curvearrowright$$



$$\frac{\overline{AB}}{\sin 40^\circ} = \frac{\overline{AP}}{\sin 20^\circ} = \frac{\overline{BP}}{\sin 120^\circ}$$

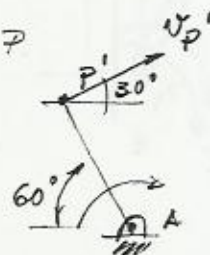
$$\overline{AB} = 250 \text{ mm}$$

$$\overline{AP} = 133 \text{ mm}$$

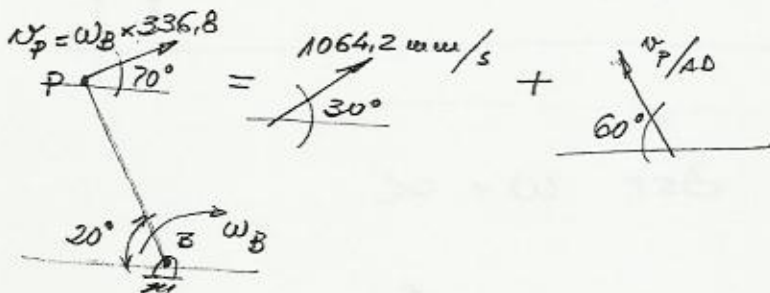
$$\overline{BP} = 336,8 \text{ mm}$$

P' - ponto da barra AD coincidente com P

$$v_{P'} = \omega_A \times AP = 8 \times 133 = 1064,2 \text{ mm/s}$$



$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/AD}$$



Comp. \rightarrow :

$$\omega_B \times 336,8 \cos 70^\circ = 1064,2 \cos 30^\circ - v_{P/AD} \cos 60^\circ$$

$$115,19 \omega_B = 921,61 - v_{P/AD} \cos 60^\circ$$

$$\omega_B = \frac{921,61 - v_{P/AD} \cos 60^\circ}{115,19}$$

$$\omega_B = 8 - v_{P/AD} \times 0,0043$$

Comp. \uparrow :

$$\omega_B \times 336,8 \sin 70^\circ = 1064,2 \sin 30^\circ + v_{P/AD} \sin 60^\circ$$

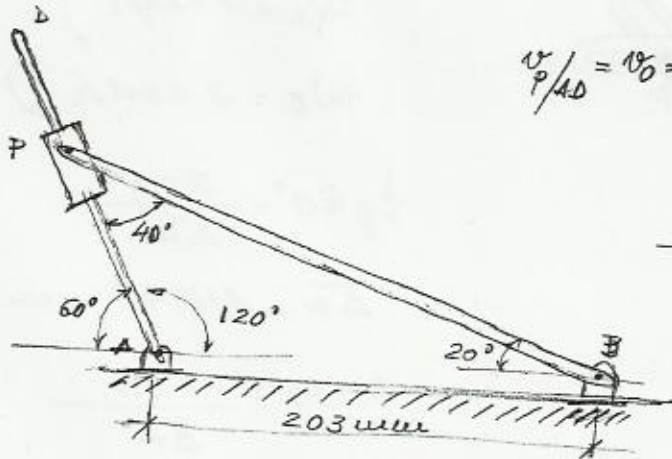
$$(8 - v_{P/AD} \times 0,0043) \times 336,8 \sin 70^\circ = 532,1 + 0,87 v_{P/AD}$$

$$2.531,91 - 1,36 v_{P/AD} = 532,1 + 0,87 v_{P/AD}$$

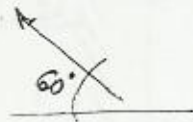
$v_{P/AD} = 896,41 \text{ mm/s}$

$\omega_B = 4,15 \text{ rad/s}$

15.117-



$$v_{P/AD} = v_0 = 305 \text{ mm/s}$$



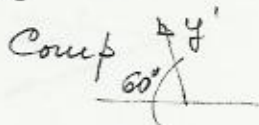
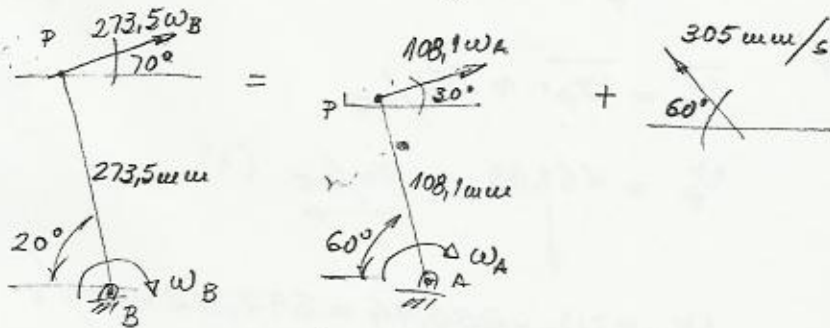
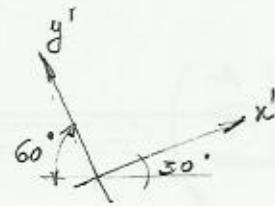
$$\frac{203}{\sin 40^\circ} = \frac{\overline{BP}}{\sin 120^\circ} = \frac{\overline{AP}}{\sin 20^\circ}$$

$$\overline{AP} = 108,01 \text{ mm}$$

$$\overline{BP} = 273,50 \text{ mm}$$

P': ponto de barra \overline{AD} coincidente com P.

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/AD}$$



$$273,5 \omega_B \sin 40^\circ = 305$$

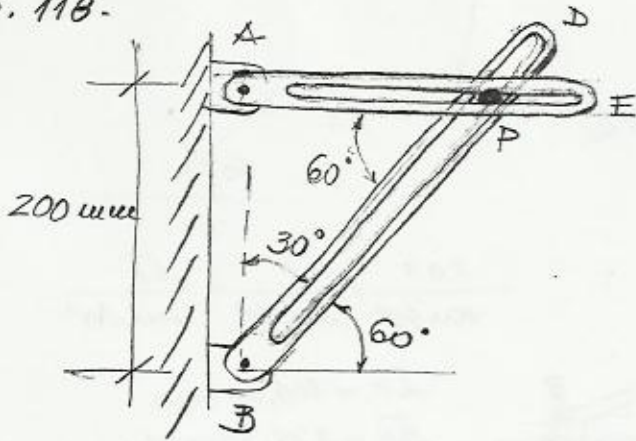
$$\omega_B = 1,73 \text{ rad/s}$$



$$273,5 \omega_B \cos 40^\circ = 108,1 \omega_A$$

$$\omega_A = 3,36 \text{ rad/s}$$

15.118.



$$\omega_A = 4 \text{ rad/s}$$

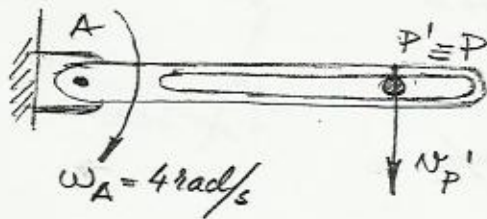
$$\omega_B = 3 \text{ rad/s}$$

$$\tan 60^\circ = \frac{200}{\overline{AP}}$$

$$\overline{AP} = 115,47 \text{ mm}$$

$$\cot 30^\circ = \frac{200}{\overline{BP}}$$

$$\overline{BP} = 230,94 \text{ mm}$$



$$v_{P'} = \omega_A \times 115,47 = 461,88 \text{ mm/s}$$

$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/AE}$$

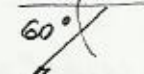
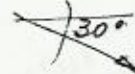
$$v_P = 461,88 + v_{P/AE} \quad (1)$$

$$v_{P''} = \omega_B \times 230,94 = 692,82 \text{ mm/s}$$



$$\vec{v}_P = \vec{v}_{P''} + \vec{v}_{P/BD}$$

$$v_P = 692,82 + v_{P/BD} \quad (2)$$



IGUALANDO AS EQS. (1) E (2).

$$461,88 + v_{P/AE} = 692,82 + v_{P/BD}$$

Comp. \uparrow :

$$-461,88 = -692,82 \sin 30^\circ - v_{P/BD} \sin 60^\circ$$

$$v_{P/BD} = 133,33 \text{ mm/s}$$

Comp. \rightarrow :

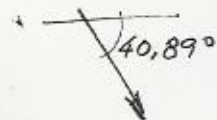
$$v_{P/AE} = 692,82 \cos 30^\circ - v_{P/BD} \cos 60^\circ$$

$$v_{P/AE} = 533,33 \text{ mm/s}$$

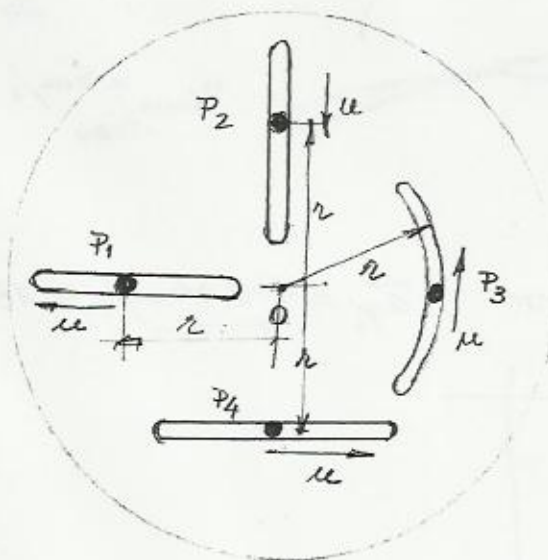
$$\vec{v}_P = \vec{v}_{P'} + \vec{v}_{P/AE}$$

$$v_P = 461,88 + 533,33$$

$$v_P = 705,53 \text{ mm/s}$$



15.120 -



ω ↻

$\omega = cte$

P' — ponto do disco coincidente c/cada ponto P_1, P_2, P_3, P_4

$$a_{P'} = r\omega^2$$

ACELERAÇÃO DO PINO EM RELAÇÃO AO DISCO:

$$P/P_1, P_2, P_4: a_{P/D} = 0$$

$$P/P_3: a_{P/D} = \frac{u^2}{r}$$

$a_C = 2\omega u$ P/ CADA PINO,

DIREÇÃO PERPENDICULAR A u , SENTIDO DE ROTACÃO DE ω (REGRAS DA MÃO DIREITA)

$$\vec{a} = \vec{a}_{P'} + \vec{a}_{P/D} + \vec{a}_C$$

$$\vec{a}_1 = [r\omega^2 \rightarrow] + [2\omega u \downarrow]$$

$$\vec{a}_2 = [r\omega^2 \downarrow] + [2\omega u \rightarrow]$$

$$\vec{a}_3 = [r\omega^2 \leftarrow] + \left[\frac{u^2}{r} \leftarrow \right] + [2\omega u \uparrow]$$

$$\vec{a}_4 = [r\omega^2 \uparrow] + [2\omega u \leftarrow]$$

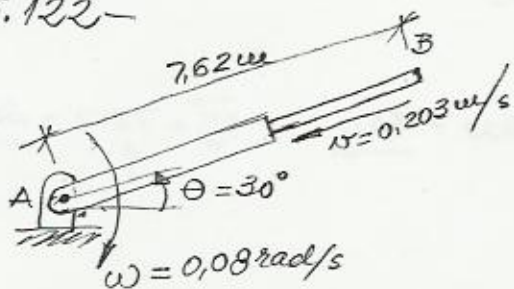
$$\vec{a}_1 = r\omega^2 \vec{i} - 2\omega u \vec{j}$$

$$\vec{a}_2 = 2\omega u \vec{i} - r\omega^2 \vec{j}$$

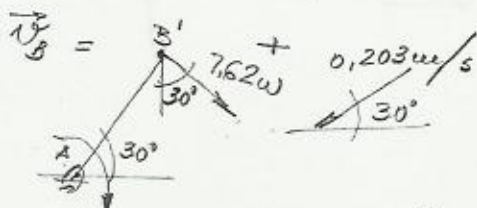
$$\vec{a}_3 = -(r\omega^2 + \frac{u^2}{r} + 2\omega u) \vec{i}$$

$$\vec{a}_4 = (r\omega^2 + 2\omega u) \vec{j}$$

15.122 -



$$\vec{v}_B = \vec{v}_{B'} + \vec{v}_{B/AB}$$



$$v_{Bx} = 0.08 \times 7.62 \cos 30^\circ - 0.203 \cos 30^\circ$$

$$v_{By} = -0.08 \times 7.62 \sin 30^\circ - 0.203 \sin 30^\circ$$

$$v_{Bx} = 0.129\text{ m/s} \rightarrow$$

$$v_{By} = 0.629\text{ m/s} \downarrow$$

$$v_B = 0.64\text{ m/s}$$

$$78.41^\circ$$

$$\vec{a}_B = \vec{a}_{B'} + \vec{a}_{B/AB} + \vec{a}_C$$

$$\vec{a}_B = \omega^2 \cdot 7.62 + 0 + 2\omega \times v$$

$$a_{Bx} = -0.08^2 \times 7.62 \cos 30^\circ - 2 \times 0.08 \times 0.203 \cos 30^\circ$$

$$a_{Bx} = -0.059\text{ m/s}^2 \leftarrow$$

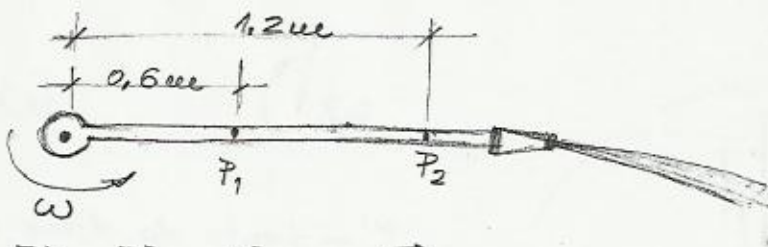
$$a_{By} = -0.08^2 \times 7.62 \sin 30^\circ + 2 \times 0.08 \times 0.203 \sin 30^\circ$$

$$a_{By} = 0.0037\text{ m/s}^2 \uparrow$$

$$a_B = 0.0586\text{ m/s}^2$$

$$3.59^\circ$$

15.124-

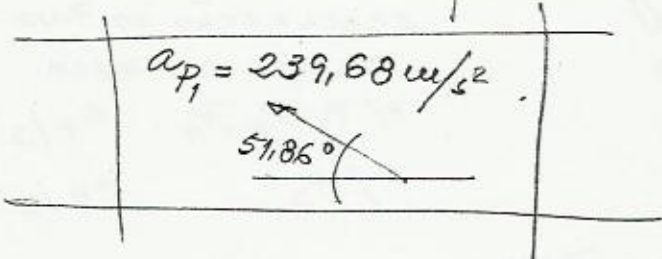


$$\omega = 150 \text{ rpm} = \frac{150\pi}{30} = 5\pi \text{ rad/s}$$

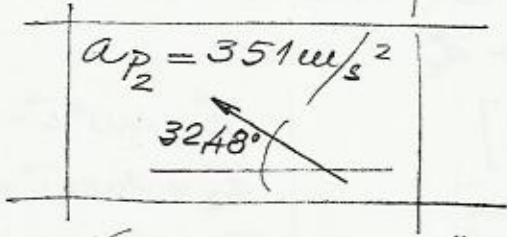
$$v_{\text{LGA/TUBO}} = 6 \text{ m/s}$$

$$\vec{a}_p = \vec{a}_{p1} + \vec{a}_{p/TUBO} + \vec{a}_c$$

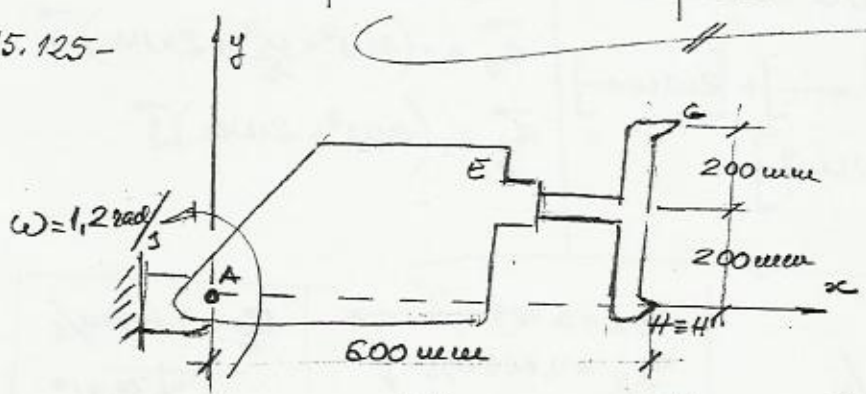
a) $\vec{a}_{p1} = \omega^2 \times 0,6 + 0 + 2\omega \times v$ $\vec{a}_{p1} = 148,04 \text{ m/s}^2 + 188,50 \text{ m/s}^2$



b) $\vec{a}_{p2} = \omega^2 \times 1,2 + 0 + 2\omega \times v \therefore \vec{a}_{p2} = 296,08 \text{ m/s}^2 + 188,50 \text{ m/s}^2$



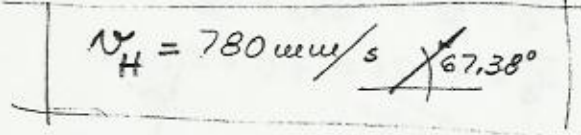
15.125-



$$v_{H/AE} = 300 \text{ mm/s}$$

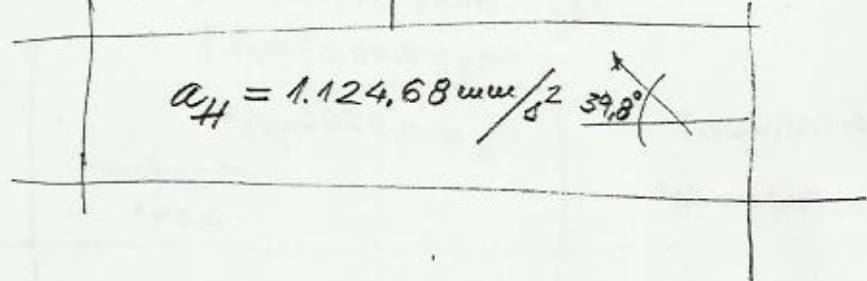
$$a_{H/AE} = 0$$

$$\vec{v}_H = \vec{v}_{H'} + \vec{v}_{H/AE} \quad \vec{v}_H = 1,2 \times 600 + 300 \quad \vec{v}_H = 720 + 300$$

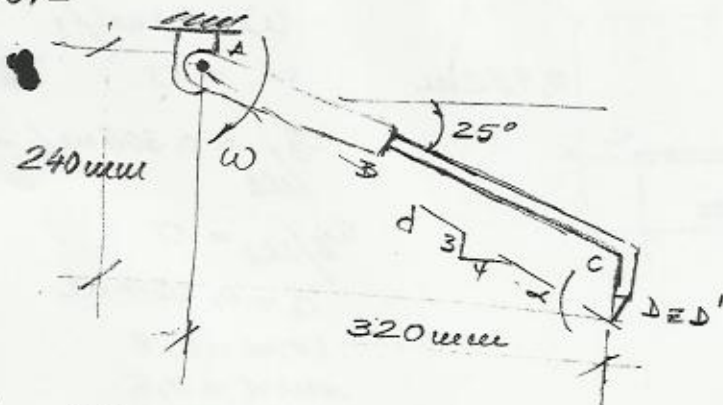


$$\vec{a}_H = \vec{a}_{H'} + \vec{a}_{H/AE} + \vec{a}_c \quad \vec{a}_H = 1,2^2 \times 600 + 0 + 2 \times 1,2 \times 300$$

$$\vec{a}_H = 864 + 720$$



15.127-



$$\omega = 1,5 \text{ rad/s}$$

$$\alpha = 0$$

$$v_{D/AB} = 250 \text{ mm/s} \quad \swarrow 25^\circ$$

$$a_{D/AB} = 0$$

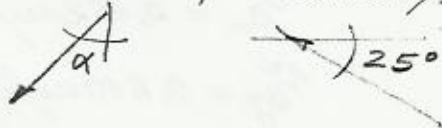
$$d = 400 \text{ mm}$$

$$\cos \alpha = 0,8$$

$$\sin \alpha = 0,6$$

$$a) \quad \vec{v}_D = \vec{v}_{D'} + \vec{v}_{D/AB}$$

$$\vec{v}_D = \omega \times 400 + 250 \text{ mm/s}$$



$$v_{Dx} = -1,5 \times 400 \sin \alpha - 250 \cos 25^\circ$$

$$v_{Dx} = 586,58 \text{ mm/s} \quad \leftarrow$$

$$v_{Dy} = -1,5 \times 400 \cos \alpha + 250 \sin 25^\circ$$

$$v_{Dy} = 374,35 \text{ mm/s} \quad \downarrow$$

$$v_D = 695,85 \text{ mm/s} \quad \swarrow 32,5^\circ$$

$$b) \quad \vec{a}_D = \vec{a}_{D'} + \vec{a}_{D/AB} + \vec{a}_C$$

$$\vec{a}_D = \omega^2 \times 400 + 0 + 2\omega \times 250$$

$$a_{Dx} = -1,5^2 \times 400 \cos \alpha + 2 \times 1,5 \times 250 \sin 25^\circ$$

$$a_{Dx} = 403,04 \text{ mm/s}^2 \quad \leftarrow$$

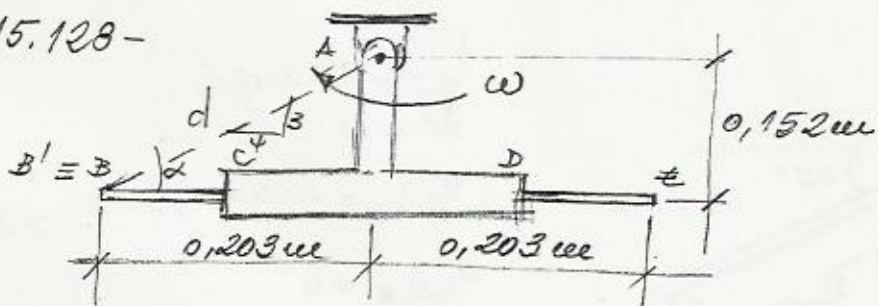
$$a_{Dy} = 1,5^2 \times 400 \sin \alpha + 2 \times 1,5 \times 250 \cos 25^\circ$$

$$a_{Dy} = 1.219,73 \text{ mm/s}^2 \quad \uparrow$$

$$a_D = 1.284,59 \text{ mm/s}^2$$

$$74,71^\circ$$

15.128-



$$\omega = 3 \text{ rad/s}$$

$$\alpha = 0$$

$$v_{B/ACS} = 0,305 \text{ m/s} \rightarrow$$

$$a_{B/ACS} = 0$$

$$d = 0,254 \text{ m}$$

$$\cos \alpha = 0,8$$

$$\sin \alpha = 0,6$$

$$a) \vec{v}_B = \vec{v}_{B'} + \vec{v}_{B/ACS}$$

$$\vec{v}_B = \omega \times 0,254 + 0,305$$



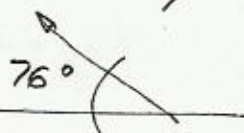
$$v_{Bx} = -3 \times 0,254 \sin \alpha + 0,305$$

$$v_{By} = 3 \times 0,254 \cos \alpha$$

$$v_{Bx} = 0,152 \text{ m/s} \leftarrow$$

$$v_{By} = 0,610 \text{ m/s} \uparrow$$

$$v_B = 0,628 \text{ m/s}$$



$$b) \vec{a}_B = \vec{a}_{B'} + \vec{a}_{B/ACS} + \vec{a}_C$$

$$\vec{a}_B = \omega^2 \times 0,254 + 0 + 2 \times \omega \times 0,305$$



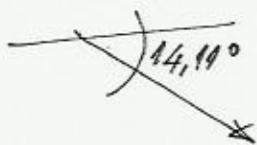
$$a_{Bx} = 3^2 \times 0,254 \times \cos \alpha$$

$$a_{By} = 3^2 \times 0,254 \times \sin \alpha - 2 \times 3 \times 0,305$$

$$a_{Bx} = 1,83 \text{ m/s}^2 \rightarrow$$

$$a_{By} = 0,46 \text{ m/s}^2 \downarrow$$

$$a_B = 1,89 \text{ m/s}^2$$

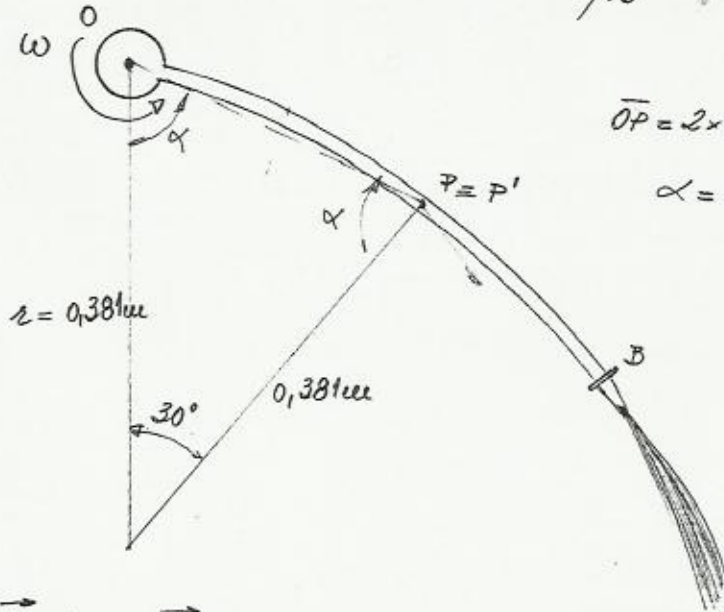


15.134-

$\alpha = 0$

$\omega = 120 \text{ rpm} = \frac{120 \times \pi}{30} = 4\pi \text{ rad/s}$

$v = v_P / 0.8 = 12.2 \text{ m/s}$



$\overline{OP} = 2 \times 0.381 \sin \frac{30^\circ}{2} = 0.197 \text{ m}$

$\alpha = 75^\circ$

$\vec{a}_P = \vec{a}_{P'} + \vec{a}_{P/OB} + \vec{a}_C$

$\vec{a}_P = \omega^2 \overline{OP} + \frac{v^2}{r} + 2\omega v$

$a_{P_n} = -(4\pi)^2 \times 0.197 \times \sin 75^\circ - \frac{(12.2)^2}{0.381} \sin 30^\circ + 2 \times 4\pi \times 12.2 \sin 30^\circ$

$a_{P_n} = -72.07 \text{ m/s}^2$

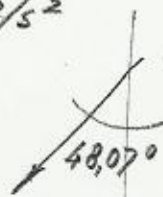
$a_{P_n} = 72.07 \text{ m/s}^2$

$a_{P_y} = (4\pi)^2 \times 0.197 \cos 75^\circ - \frac{(12.2)^2}{0.381} \cos 30^\circ + 2 \times 4\pi \times 12.2 \cos 30^\circ$

$a_{P_y} = -64.73 \text{ m/s}^2$

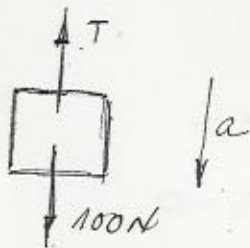
$a_{P_y} = 64.73 \text{ m/s}^2$

$a_P = 96.87 \text{ m/s}^2$



Cap. 12

12.3- BALANCA DE MOLLA -



ELEVADOR EM MOVIMENTO

$$T = 90 \text{ N}$$

$$R = m a$$

$$100 - T = \frac{100}{g} \cdot a$$

$$100 - 90 = \frac{100}{g} \cdot a$$

$$a = 0,98 \text{ m/s}^2$$

BALANCA DE ALAVANCA



$$\sum M_O = 0$$

$$c T_1 = b T_2$$

$$T_1 = \frac{b}{c} T_2$$

$$T_2 = \frac{c}{b} T_1$$

ELEVADOR EM REPOUSO



$$T_1 = 100 \text{ N}$$

$$T_2 = W$$

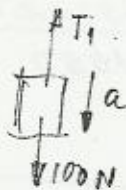
$$T_2 = \frac{c}{b} T_1$$

$$T_2 = \frac{c}{b} \times 100$$

CONSIDERANDO-SE OS

BRACOS IGUAIS $\Rightarrow T_2 = 100 \text{ N}$ $W = 100 \text{ N}$.

ELEVADOR EM MOVIMENTO.



$$100 - T_1 = \frac{100}{g} \cdot a$$

$$T_1 = 100 \left(1 - \frac{a}{g} \right)$$

$$T_2 = \frac{c}{b} T_1$$

$$W' = \frac{c}{b} 100 \text{ N}$$

CONSIDERANDO-SE BRACOS IGUAIS.

$$W' = 100 \text{ N}$$

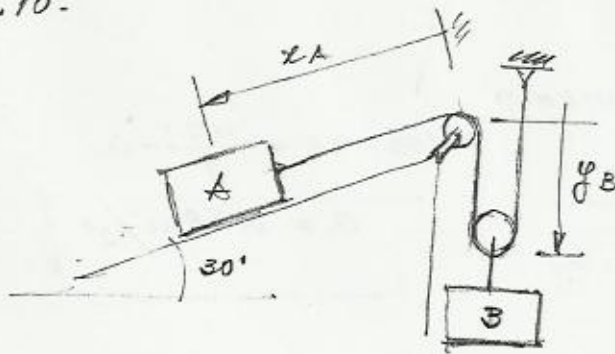
$$W' - T_2 = \frac{W'}{g} a$$

$$W' - \frac{W'}{g} a = T_2$$

$$T_2 = W' \left(1 - \frac{a}{g} \right)$$

$$\frac{c}{b} \left[100 \left(1 - \frac{a}{g} \right) \right] = W' \left(1 - \frac{a}{g} \right)$$

12.10.



$$F_A = 890 \text{ N}$$

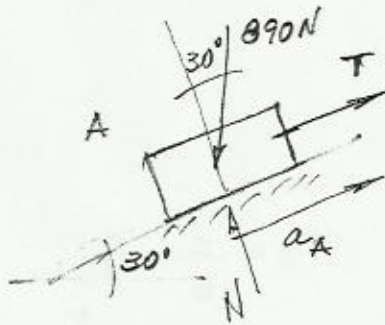
$$F_B = 1557 \text{ N}$$

$$x_A + 2y_B = L$$

$$\frac{d}{dt} v_A + 2v_B = 0$$

$$\frac{d}{dt} a_A = -2a_B$$

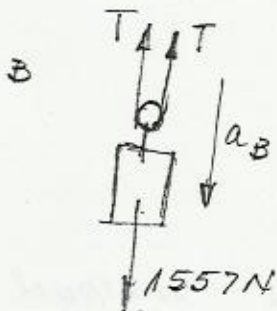
$$|\vec{a}_A| = 2|\vec{a}_B|$$



$$T - 890 \sin 30^\circ = \frac{890}{g} \cdot a_A$$

$$|\vec{a}_A| = 2|\vec{a}_B|$$

$$T - 890 \sin 30^\circ = 2 \times \frac{890}{g} a_B \quad (1)$$



$$1557 - 2T = \frac{1557}{g} a_B$$

$$a_B = \frac{(1557 - 2T) \times g}{1557} \quad (2)$$

Subst. (2) in (1):

$$T - 890 \sin 30^\circ = 2 \times \frac{890}{g} \times \frac{(1557 - 2T) \times g}{1557}$$

$$T - 445 = 1780 - 2,29T$$

$$3,29T = 1780 + 445$$

$$T = 677 \text{ N}$$

$$a_B = \frac{(1557 - 2T) \times g}{1557}$$

$$a_B = 1,28 \text{ m/s}^2$$

$$a_A = 2a_B$$

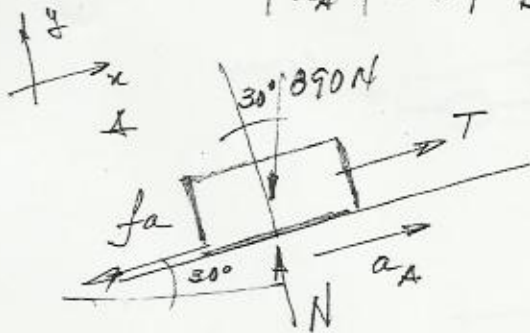
$$a_A = 2,56 \text{ m/s}^2$$

12.11 - DO PROBLEMA ANTERIOR

$$\vec{a}_A = -2\vec{a}_B$$

$$|\vec{a}_A| = 2|\vec{a}_B|$$

$$\mu_c = 0,20$$



CORPO A

$$\sum F_y = 0$$

$$N = 890 \cos 30^\circ$$

$$f_a = \mu_c N \quad f_a = 0,2 \times 890 \cos 30^\circ$$

$$f_a = 154,15 \text{ N}$$

$$\sum F_x = m a$$

$$T - f_a - 890 \sin 30^\circ = \frac{890}{g} a_A$$

$$T - 599,15 = \frac{890}{g} a_A$$

$$a_A = 2a_B$$

$$T - 599,15 = \frac{1780}{g} a_B \quad (1)$$

P/ CORPO B

$$1557 - 2T = \frac{1557}{g} a_B$$

$$a_B = \frac{1557 - 2T}{1557} g \quad (2)$$

subst. (2) em (1)

$$T - 599,15 = \frac{1780}{g} \frac{1557 - 2T}{1557} g$$

$$T - 599,15 = 1780 - 2,29T$$

$$3,29T = 2.379,15$$

$$T = 1.040,54 \text{ N}$$

Subst. T em (2)

$$a_B = -3,30 \text{ m/s}^2$$

$$a_B = 3,30 \text{ m/s}^2$$

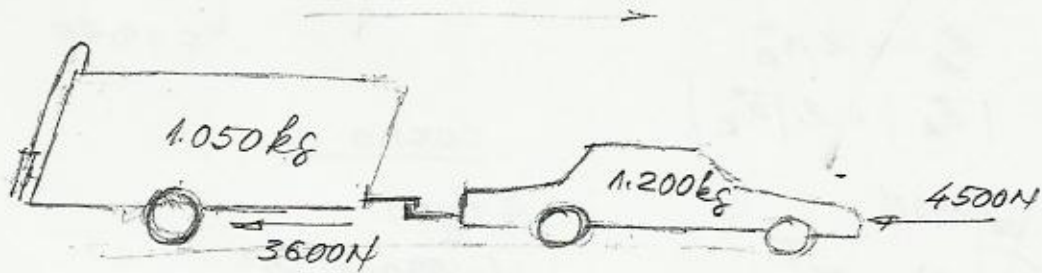
$$a_A = -2a_B$$

$$a_A = -2(-3,3)$$

$$a = 6,6 \text{ m/s}^2$$

30°

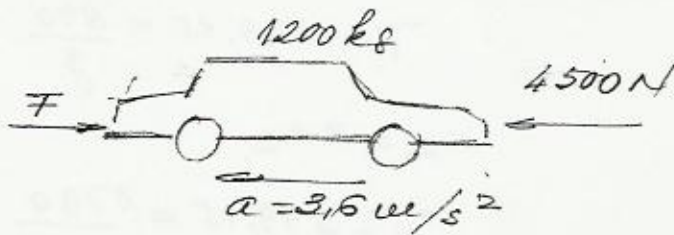
12.12-



$$R = m a$$

$$8100 = 2250 a$$

$$a = 3,6 \text{ m/s}^2$$



$$4500 - F = 1200 \times 3,6$$

$$F = 180 \text{ N}$$

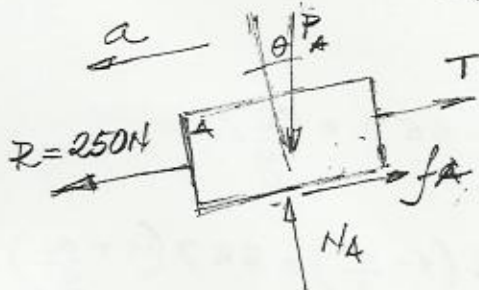
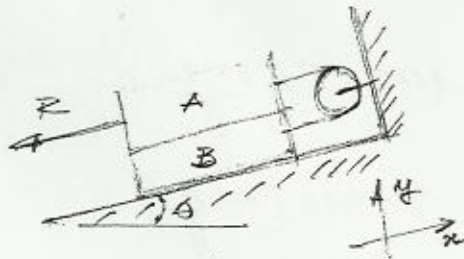
12.14-

$$\theta = 30^\circ \quad \mu_e = 0,15$$

$$m_A = 30 \text{ kg} \quad \mu_c = 0,10$$

$$m_B = 15 \text{ kg} \quad \text{Det. } a_A$$

$$T$$



CORPO A -

$$\Sigma F_y = 0$$

$$P_A \cos 30^\circ = N_A \quad N_A = 30g \cos 30^\circ$$

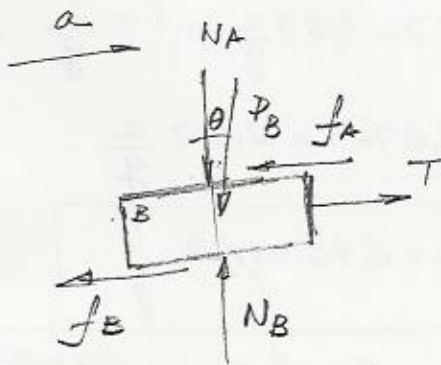
$$f_A = 0,1 \times 30g \cos 30^\circ$$

$$f_A = 3g \cos 30^\circ$$

$$\Sigma F_x = ma$$

$$250 = T + 30g \sin 30^\circ - f_A = 30 \times a$$

$$371,66 - T = 30a \quad (1)$$



CORPO B -

$$\Sigma F_y = 0$$

$$N_B = N_A + P_B \cos 30^\circ$$

$$N_B = 30g \cos 30^\circ + 15g \cos 30^\circ$$

$$N_B = 45g \cos 30^\circ$$

$$f_B = 0,1 \times 45g \cos 30^\circ = 4,5g \cos 30^\circ$$

$$\Sigma F_x = ma$$

$$T - P_B \sin 30^\circ - f_A - f_B = 15a$$

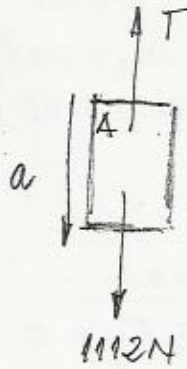
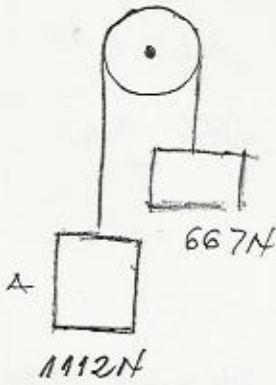
$$T - 15g \sin 30^\circ - 3g \cos 30^\circ - 4,5g \cos 30^\circ = 15a$$

$$T - 137,29 = 15a \quad (2)$$

$$\begin{cases} 371,66 - T = 30a \\ -137,29 + T = 15a \end{cases}$$

$$a = 5,21 \text{ m/s}^2 \quad \swarrow 30^\circ$$

$$T = 215,42 \text{ N}$$



$$c) 1112 - T = \frac{1112 \cdot a}{g}$$

$$T = 1112 \left(1 - \frac{a}{g}\right)$$

$$T - 667 = \frac{667 \cdot a}{g}$$

$$1112 \left(1 - \frac{a}{g}\right) = 667 \left(1 + \frac{a}{g}\right)$$

$$\frac{1112}{667} \left(1 - \frac{a}{g}\right) = 1 + \frac{a}{g}$$

$$1,67 - 1,67 \frac{a}{g} = 1 + \frac{a}{g}$$

$$1,67 - 1 = 2,67 \frac{a}{g}$$

$$a = 2,45 \text{ m/s}^2 \downarrow$$

$$t = 2 \text{ s}$$

$$b) v = v_0 + at$$

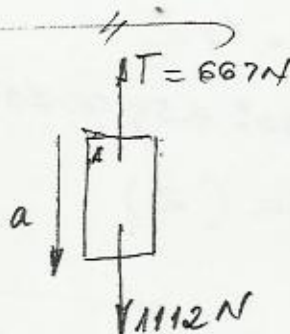
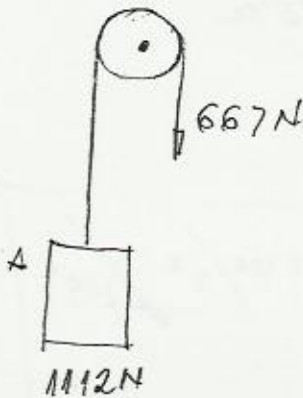
$$v = 2,45 \times 2$$

$$v = 4,90 \text{ m/s} \downarrow$$

$$c) v^2 = v_0^2 + 2a \Delta x \quad \Delta x = 2,44 \text{ m}$$

$$v^2 = 2 \times 2,45 \times 2,44$$

$$v = 3,46 \text{ m/s} \downarrow$$



$$a) 1112 - 667 = \frac{1112 \cdot a}{g}$$

$$a = 3,93 \text{ m/s}^2 \downarrow$$

$$b) t = 2 \text{ s}$$

$$v = v_0 + at$$

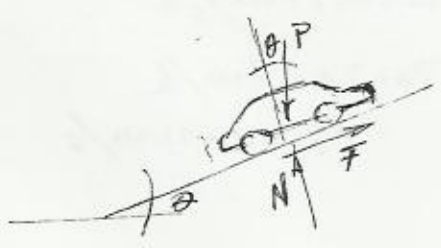
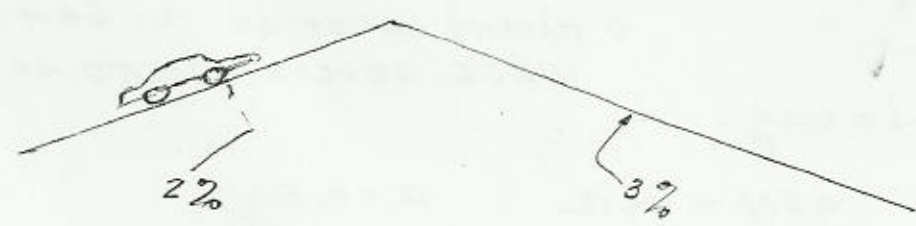
$$v = 3,93 \times 2$$

$$v = 7,86 \text{ m/s} \downarrow$$

$$c) \Delta x = 2,44 \text{ m}$$

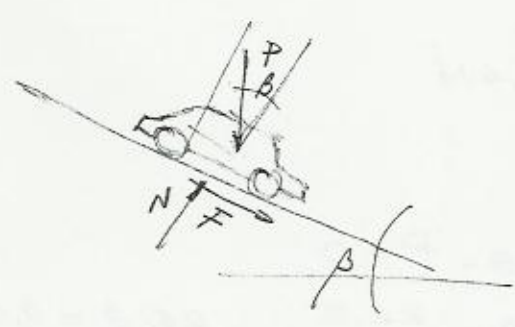
$$v^2 = 2 \times 3,93 \times 2,44$$

$$v = 4,38 \text{ m/s} \downarrow$$



F é a força do motor
 $\text{tg } \theta = 0,02 \quad \theta = 1,15^\circ$

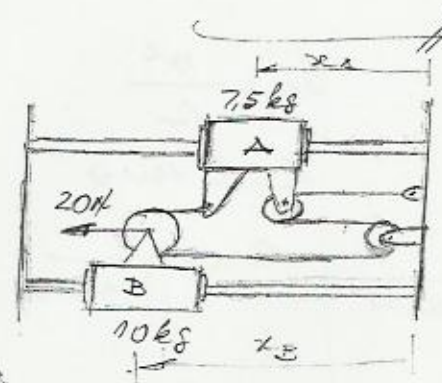
$F - P \text{sen } \theta = 0 \Rightarrow v = \text{cte.}$
 $F = mg \text{sen } \theta \quad F = 0,02 mg$



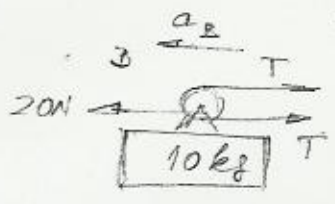
$\text{tg } \beta = 0,03 \quad \beta = 1,71^\circ$
 $F + P \text{sen } \beta = m a$
 $0,02 mg + 0,03 mg = m a$

$a = 0,05 g$
 $a = 0,49 \text{ m/s}^2$

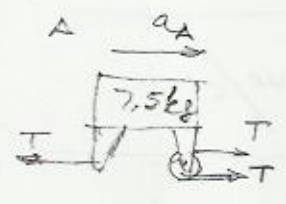
12.16



$x_B + 2x_A + (x_B - 2x_A) = L$
 $2x_B + x_A = L$
 $\frac{d^2}{dt^2} \quad 2a_B + a_A = 0 \quad \vec{a}_A = -2\vec{a}_B$
 $|\vec{a}_A| = 2|\vec{a}_B|$



$20 - 2T = 10 a_B$
 $20 - 3T = 10 a_B$
 $20 = 40 a_B$
 $a_B = \frac{1}{2} \text{ m/s}^2$

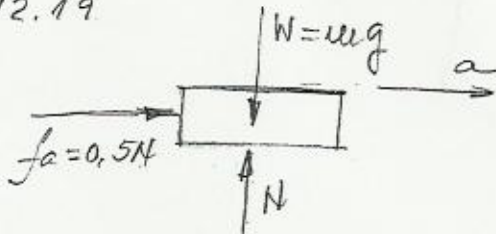


$2T - T = 7,5 a_A$
 $T = 7,5 a_A$
 $T = 15 a_B$
 $a_A = 1 \text{ m/s}^2$

$v_A = a_A t$
 $v_A = 1 \times 1,2$

$v_A = 1,2 \text{ m/s}$
 $v_B = 0,6 \text{ m/s}$

12.19



$$\sum F_y = 0 \quad N = mg$$

$$\sum F_x = ma \quad 0,5N = ma$$

$$0,5mg = ma$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$0 = 20,11^2 - 2 \times 4,91 \Delta x$$

$$\Delta x = 41,19 \text{ m}$$

CONSIDERANDO-SE QUE A CARGA ESTEJA NA IMINÊNCIA DE MOVIMENTO, CALCULAREMOS O MÁXIMO VALOR DE a SEM QUE HAJA DESLIZAMENTO DA CARGA.

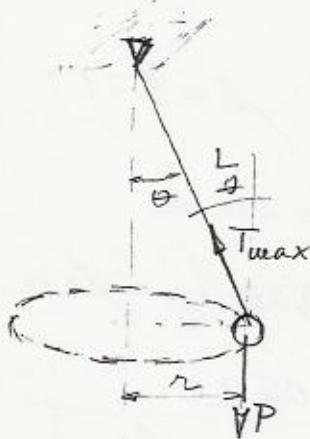
$$a = 0,5g$$

$$a = 4,91 \text{ m/s}^2$$

$$v_0 = 72,4 \text{ km/h}$$

$$v_0 = 20,11 \text{ m/s}$$

12.35



$$P = 22,2 \text{ N}$$

$$L = 1,22 \text{ m}$$

$$T_{\text{max}} = 53,4 \text{ N}$$

$$\sum F_y = 0$$

$$T_{\text{max}} \cos \theta - P = 0$$

$$\cos \theta = \frac{22,2}{53,4}$$

$$\cos \theta = 0,41$$

$$\theta = 65,43^\circ$$

$$\sum F_x = m a_m$$

$$T_{\text{max}} \sin \theta = \frac{22,2}{g} \cdot a_m$$

$$53,4 \times \sin 65,43^\circ = \frac{22,2}{g} \cdot \frac{v^2}{1,22 \sin 65,43^\circ}$$

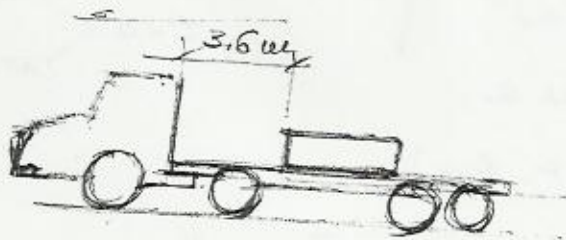
$$a_m = \frac{v^2}{r}$$

$$r = L \sin \theta$$

$$a_m = \frac{v^2}{L \sin \theta}$$

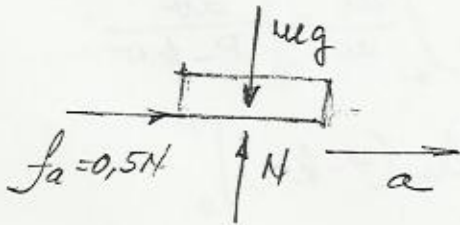
$$v = 4,88 \text{ m/s}$$

12.19



$$\mu_e = 0,50$$

$$\mu_c = 0,40$$



$$0,5N = m a$$

$$0,5 \mu_e g = \mu_e a$$

$$a = 0,5g \quad a = 4,91 \text{ m/s}^2$$

a - aceleração linear, o corpo está prestes ao deslizamento.

Cinemática:

$$v^2 = v_0^2 + 2a \Delta x$$

$$v_0 = 72,4 \text{ km/h} = 20,11 \text{ m/s}$$

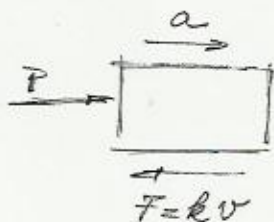
$$v = 0$$

$$0^2 = \left(\frac{72,4}{3,6} \right)^2 - 2 \times 4,91 \Delta x$$

$$\Delta x = 41,18 \text{ m}$$

12.20 -

12.26 -



$$\sum F_x = ma$$

$$P - kv = ma$$

$$a = \frac{1}{m} (P - kv)$$

$$t=0 \begin{cases} x=0 \\ v=0 \end{cases}$$

$$a = \frac{dv}{dt} = \frac{1}{m} (P - kv) = \frac{dv}{dt} \quad \int_0^t \frac{dt}{m} = \int_0^v \frac{dv}{P - kv}$$

$$\frac{1}{m} t = -\frac{1}{k} \int_0^v \frac{-k dv}{P - kv} \Rightarrow \frac{t}{m} = -\frac{1}{k} \ln (P - kv) \Big|_0^v$$

$$-\frac{kt}{m} = \ln (P - kv) - \ln P \Rightarrow -\frac{kt}{m} = \ln \frac{P - kv}{P}$$

$$\frac{P - kv}{P} = e^{-\frac{kt}{m}} \quad P - kv = P e^{-\frac{kt}{m}}$$

$$kv = P \left(1 - e^{-\frac{kt}{m}} \right)$$

$$v = \frac{P}{k} \left(1 - e^{-\frac{kt}{m}} \right)$$

$$v = \frac{dx}{dt}$$

$$\int e^u du = e^u$$

$$u = e^{-\frac{kt}{m}} \quad du = -\frac{k}{m} e^{-\frac{kt}{m}} dt$$

$$\int dx = \int v dt$$

$$\int_0^x dx = \frac{P}{k} \int_0^t \left(1 - e^{-\frac{kt}{m}} \right) dt$$

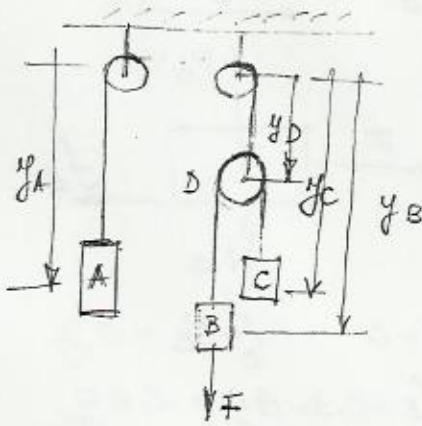
$$\int_0^x dx = \frac{P}{k} \left[\int_0^t dt + \frac{m}{k} \int_0^t -\frac{k}{m} e^{-\frac{kt}{m}} dt \right]$$

$$x = \frac{P}{k} \left[t + \frac{m}{k} e^{-\frac{kt}{m}} \Big|_0^t \right]$$

$$x = \frac{P}{k} \left[t + \frac{m}{k} \left(e^{-\frac{kt}{m}} - 1 \right) \right]$$

$$x = \frac{P}{k} \left[t + \frac{m}{k} \left(e^{-\frac{kt}{m}} - 1 \right) \right]$$

12.28-



$$y_A + y_D = L$$

$$y_B - y_D + y_C - y_D = L'$$

$$y_B + y_C - 2y_D = L'$$

$$y_D = \frac{1}{2}(y_B + y_C) + \frac{L'}{2}$$

$$y_A + \frac{1}{2}(y_B + y_C) = L''$$

$$\frac{d^2}{dt^2} a_A + \frac{1}{2}(a_B + a_C) = 0$$

$$a_A = -\frac{1}{2}(a_B + a_C)$$

CORPO B

$$F + P_B - T = \frac{P_B}{g} \times a_B$$

$$F + 44,5 - T = \frac{44,5}{g} \times 1,22$$

$$F - T = -38,97 \quad (1)$$

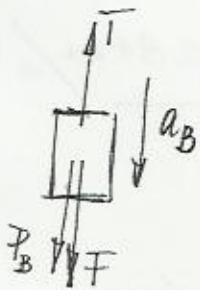
CORPO B PERCORRE
2,44 m em 2 s.

$$x_B = 0 + \frac{1}{2} a_B t^2$$

$$2,44 = \frac{1}{2} a_B \times 2^2$$

$$a_B = 1,22 \text{ m/s}^2$$

$$P_B = 44,5 \text{ N}$$



CORPO A

$$P_A = 89 \text{ N}$$

$$2T - 89 = \frac{89}{g} \times a_A$$

$$2T - 89 = \frac{89}{g} \left(+0,61 + \frac{a_C}{2} \right)$$

$$2T - 94,53 = -4,54 a_C \quad (2)$$

$$a_A = -\frac{1}{2}(a_B + a_C)$$

$$a_A = +0,61 - \frac{a_C}{2}$$

$$a_B = -1,22 \text{ m/s}^2$$

P_A

CORPO C

$$P_C = 44,5 \text{ N}$$

$$T - 44,5 = \frac{44,5}{g} \times a_C$$

$$a_C = 0,22 T - 9,81 \quad (3)$$

Subst. (3) em (2):

$$2T - 94,53 = -4,54(0,22T - 9,81)$$

$$2T - 94,53 = -T + 44,54$$

$$3T = 139,07$$

$$T = 46,35 \text{ N}$$

P_C

Subst. T em (1)

$$F - 46,35 = -38,97$$

$$F = 7,39 \text{ N}$$

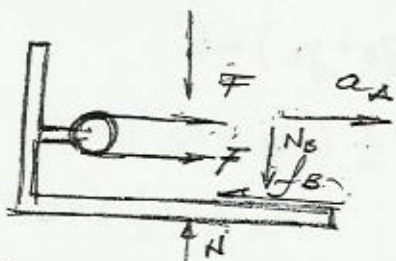
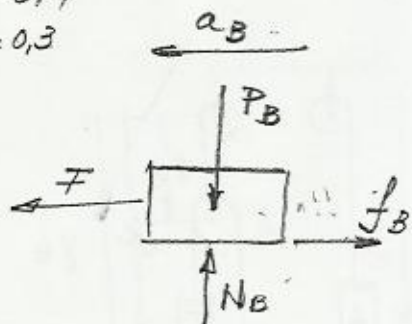
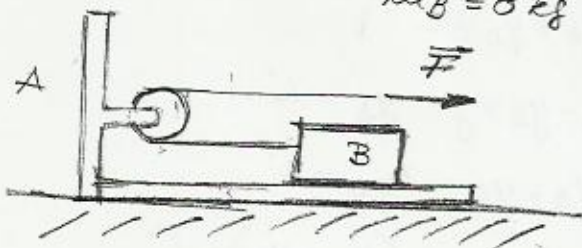
A TRACÇÃO NO CABO AD É 2T

$$T_{AD} = 2T = 92,70 \text{ N}$$

$$T_{AD} = 92,70 \text{ N}$$

12.29

$F = 30\text{N}$ $m_A = 12\text{kg}$ $\mu_c = 0,4$
 $m_B = 8\text{kg}$ $\mu_c = 0,3$



$\sum F_y = 0 \quad N_B = P_B = 8g$
 $f_B = 0,3 \times 8g = 2,4g$

$\sum F_x = ma$

$F - f_B = m_B \cdot a_B$

$30 - 2,4g = 8 \times a_B$

$a_B = 0,81 \text{ m/s}^2$

$\sum F_x = ma$

$2F - f_B = m_A a_A$

$60 - 2,4g = 12 \cdot a_A$

$a_A = 3,04 \text{ m/s}^2$

$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

$a_{B/A} = -0,81 - 3,04$

$a_{B/A} = -3,85 \text{ m/s}^2$

$a_{B/A} = 3,85 \text{ m/s}^2$

$\Delta x_{B/A} = 400 \text{ mm} = 0,4 \text{ m}$

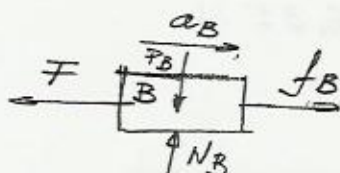
$v_{B/A}^2 = 2 \times a_{B/A} \Delta x_{B/A}$

$v_{B/A}^2 = 2 \times 3,85 \times 0,4$

$v_{B/A} = 3,08 \text{ m/s}$

12.30 - Se B NÃO ESCORREGA SOBRE O SUPORTE

$a_{B/A} = 0 \implies a_B = a_A$



$f_B = \mu_c N_B = 0,4 \times 8g = 3,2g$

$f_B - F = 8 a_B$

$3,2g - F = 8 a_B$

$a_B = a_A$

$3,2g - F = 8 a_A \quad (1)$

CORPO A.

$2F - f_B = m_A a_A$

$2F - 3,2g = 12 a_A \quad (2)$

Subst. (1) em (2).

$2F - 3,2g = \frac{3,2g - F}{8} \times 12$

$16F - 251,14 = 376,7 - 12F$

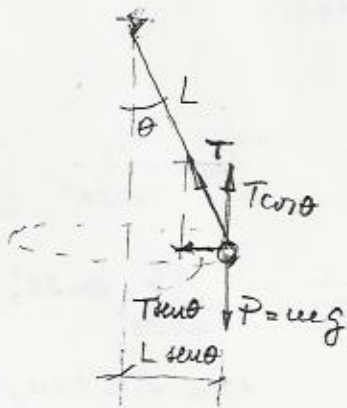
$28F = 627,84$

$F = 22,42 \text{ N}$

Subst. F em (2).

$a_A = 1,12 \text{ m/s}^2$

12.36-



$m = 3 \text{ kg}$ $L = 800 \text{ mm}$

$v = 1,2 \text{ m/s}$

$\Sigma \tau: \theta \text{ e } T.$

$\Sigma F_y = 0$

$T \cos \theta = mg = 3g \quad (1) \Rightarrow T = \frac{3g}{\cos \theta}$

$\Sigma F_x = m a_n = m \frac{v^2}{r}$

$T \sin \theta = \frac{3 \times 1,2^2}{0,8 \sin \theta} = \frac{5,4}{\sin \theta} \quad (2)$

Subst. $T = \frac{3g}{\cos \theta}$ em (2):

$\frac{3g \sin \theta}{\cos \theta} = \frac{5,4}{\sin \theta} \Rightarrow 3g \sin^2 \theta = 5,4 \cos \theta$

$\sin^2 \theta = 0,18 \cos \theta$ $1 - \cos^2 \theta = 0,18 \cos \theta$

$\cos^2 \theta + 0,18 \cos \theta - 1 = 0$

$\cos \theta = \frac{-0,18 \pm \sqrt{0,18^2 - 4 \times 1 \times (-1)}}{2}$

$\cos \theta = \frac{-0,18 \pm 2,01}{2}$

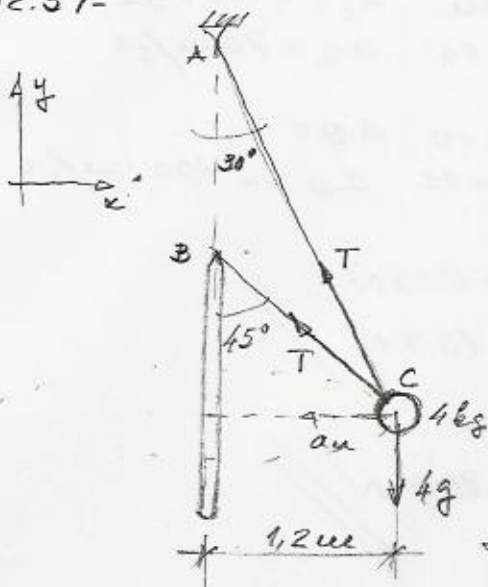
$\theta = 24,15^\circ$

Subst. θ em (1):

$T = \frac{3g}{\cos 24,15^\circ}$

$T = 32,25 \text{ N}$

12.37-



$\Sigma F_y = 0$

$T \cos 30^\circ + T \cos 45^\circ = 4g$ $T(\cos 30^\circ + \cos 45^\circ) = 4g$

$T = 24,94 \text{ N}$

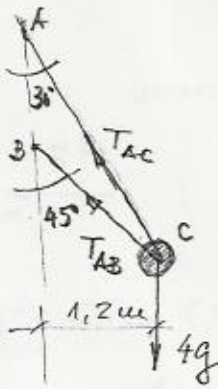
$\Sigma F_n = m a_n$

$T \sin 30^\circ + T \sin 45^\circ = 4 \times \frac{v^2}{1,2}$

$v^2 = \frac{T(\sin 30^\circ + \sin 45^\circ) \times 1,2}{4}$

$v = 3,00 \text{ m/s}$

12.39-



1ª CONSIDERAÇÃO:

$$T_{AB} = 0$$

$$\sum F_y = 0$$

$$T_{AC} \cos 30^\circ = 4g$$

$$T_{AC} = \frac{4g}{\cos 30^\circ}$$

$$\sum F_n = m a_n$$

$$T_{AC} \sin 30^\circ = 4 \times \frac{v^2}{1,2}$$

$$\frac{4g}{\cos 30^\circ} \sin 30^\circ = 4 \times \frac{v^2}{1,2}$$

$$v^2 = 1,2g \operatorname{tg} 30^\circ$$

$$v = 2,61 \text{ m/s}$$

2ª CONSIDERAÇÃO:

$$T_{AC} = 0$$

$$\sum F_y = 0$$

$$T_{AB} \cos 45^\circ = 4g$$

$$T_{AB} = \frac{4g}{\cos 45^\circ}$$

$$\sum F_n = m a_n$$

$$T_{AB} \sin 45^\circ = 4 \times \frac{v^2}{1,2}$$

$$\frac{4g}{\cos 45^\circ} \sin 45^\circ = 4 \times \frac{v^2}{1,2}$$

$$v^2 = 1,2g \operatorname{tg} 45^\circ$$

$$v = 3,43 \text{ m/s}$$

$$2,61 \text{ m/s} < v < 3,43 \text{ m/s}$$

12.65-

$$a_n = \ddot{r} - r \dot{\theta}^2$$

$$r = 25t^3 - 50t^2$$

$$\theta = t^3 - 4t$$

$$a_\theta = r \ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\dot{r} = 75t^2 - 100t$$

$$\dot{\theta} = 3t^2 - 4$$

$$\ddot{r} = 150t - 100$$

$$\ddot{\theta} = 6t$$

$$a_n = 150t - 100 - (25t^3 - 50t^2)(3t^2 - 4)^2 \quad P/t=0 \quad a_n = -100 \text{ m/s}^2$$

$$P/t=1s \quad a_n = 75 \text{ m/s}^2$$

$$a_\theta = (25t^3 - 50t^2)6t + 2(75t^2 - 100t)(3t^2 - 4) \quad P/t=0 \quad a_\theta = 0$$

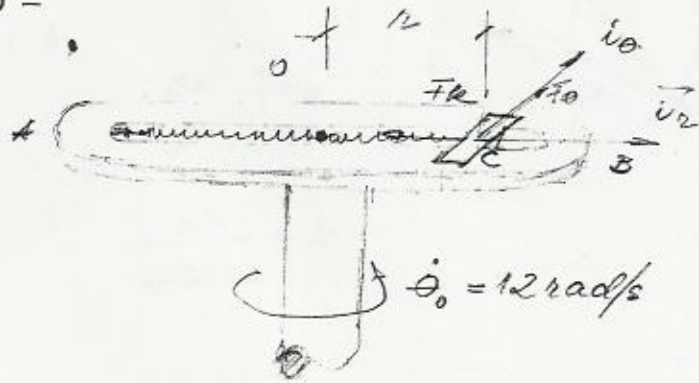
$$P/t=1s \quad a_\theta = -100 \text{ m/s}^2$$

$$m = 2 \text{ kg}$$

$$F_n = m a_n = 2a_n \quad \left\{ \begin{array}{l} t=0 \quad \overline{F_n} = -200 \text{ N} \\ t=1s \quad \overline{F_n} = 150 \text{ N} \end{array} \right. //$$

$$F_\theta = m a_\theta = 2a_\theta \quad \left\{ \begin{array}{l} t=0 \quad \overline{F_\theta} = 0 \\ t=1s \quad \overline{F_\theta} = -200 \text{ N} \end{array} \right. //$$

12.70 -



$$k = 36 \text{ N/m.}$$

$$r = 400 \text{ mm} = 0,4 \text{ m.}$$

$$\dot{\theta}_0 = 12 \text{ rad/s} = \text{cte.}$$

$$v_r = 1,8 \text{ m/s} \quad m_c = 200 \text{ g}$$

Det:

a) a_r e a_θ

b) \dot{i}

c) \bar{F}_θ

a) MOLA NÃO DEFORMADA:

$$r = 0$$

$$\Delta x \text{ (deformação da mola)} = \Delta r = 0,4 \text{ m}$$

$$\bar{F}_k = k \Delta r = 36 \times 0,4 = +14,4 \text{ N}$$

$$\bar{F}_k = -\bar{F}_r$$

$$\bar{F}_r = m_c a_r$$

$$-14,4 = 0,2 \times a_r$$

$$\boxed{a_r = -72 \text{ m/s}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = 0 \Rightarrow \dot{\theta} = \text{cte.}$$

$$a_\theta = 2 \times 1,8 \times 12$$

$$\dot{r} = v_r = 1,8 \text{ m/s}$$

$$\boxed{a_\theta = 43,2 \text{ m/s}^2}$$

b) aceleração do cursor
relativa à peça AC $\Rightarrow \dot{i}$

$$a_r = \dot{i} - r\dot{\theta}^2$$

$$\dot{i} = a_r + r\dot{\theta}^2$$

$$\dot{i} = -72 + 0,4 \times 12^2$$

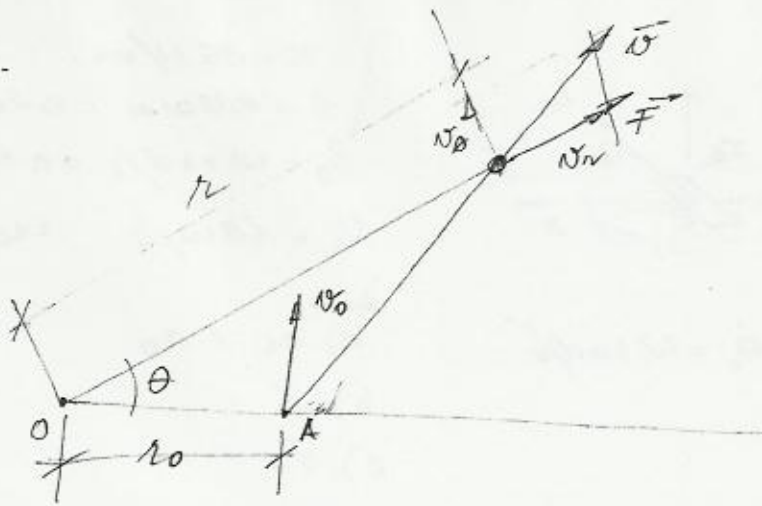
$$\boxed{\dot{i} = -14,4 \text{ m/s}^2}$$

c) $\bar{F}_\theta = m_c a_\theta$

$$\bar{F}_\theta = 0,2 \times 43,2$$

$$\boxed{\bar{F}_\theta = 8,64 \text{ N}}$$

12.74.



Det. v_θ
 v_r .

$$r = \frac{r_0}{\cos 2\theta}$$

CONSERVAÇÃO DO MOMENTO ANGULAR

$$v_\theta \cdot r = v_0 r_0$$

$$v_\theta \cdot \frac{r_0}{\cos 2\theta} = v_0 r_0$$

$$v_\theta = v_0 \cos 2\theta$$

$$\vec{v} = \dot{r} \vec{u}_r + r \dot{\theta} \vec{u}_\theta$$

$$v_\theta = r \dot{\theta} \quad v_0 \cos 2\theta = \frac{r_0}{\cos 2\theta} \dot{\theta} \quad \dot{\theta} = v_0 \frac{\cos^2 2\theta}{r_0}$$

$$r = \frac{r_0}{\cos 2\theta} = r_0 \cos^{-1} 2\theta$$

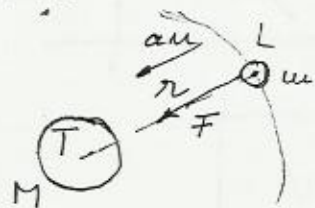
$$\dot{r} = -r_0 \cos^{-2} 2\theta (\sin 2\theta) 2\dot{\theta}$$

$$\dot{r} = \frac{r_0 \sin 2\theta}{\cos^2 2\theta} 2 \cdot \dot{\theta} = \frac{2 r_0 \sin 2\theta}{\cos^2 2\theta} v_0 \frac{\cos^2 2\theta}{r_0}$$

$$v_r = \dot{r} = 2 v_0 \sin 2\theta$$

$$v_r = 2 v_0 \sin 2\theta$$

12.78 -



$$F = \frac{GMm}{r^2}$$

$$F = \frac{gR^2 m}{r^2}$$

$$m a_n = \frac{gR^2}{r^2} m$$

$$\frac{v^2}{r} = \frac{gR^2}{r^2}$$

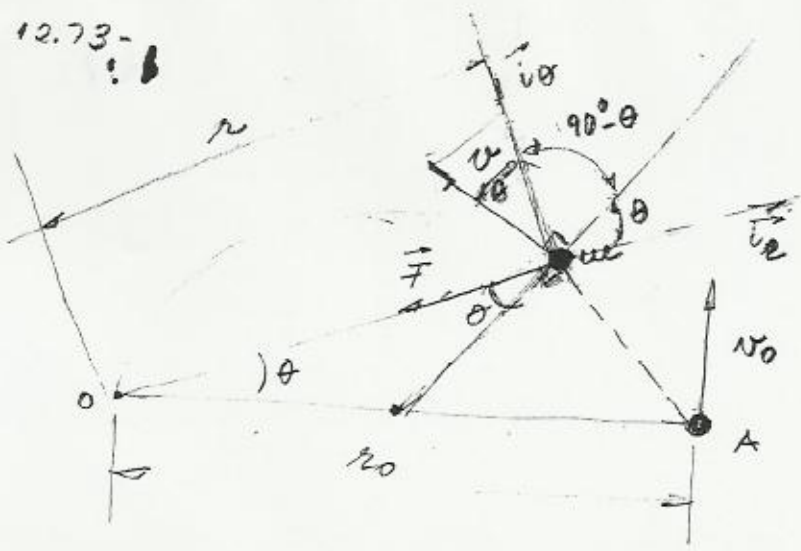
$$F = m g = \frac{GMm}{R^2} \quad GM = gR^2$$

$$v = \frac{2\pi r}{T}$$

$$\frac{2\pi r}{T^2} = \frac{gR^2}{r}$$

$$r^3 = \frac{gR^2 T^2}{4\pi^2}$$

12.73-



$$r = r_0 \cos \theta$$

$$r_0 = \frac{r}{\cos \theta}$$

$$\cos \theta = \frac{r}{r_0}$$

Mov. SOB A AÇÃO DE FORÇA CENTRAL
 - CONSERVAÇÃO DO MOMENTO ANGULAR -

$$(\vec{H}_O)_{\text{antes}} = (\vec{H}_O)_{\text{depois}}$$

$$m v_0 r_0 = m v r \cos \theta$$

$$v_0 r_0 = v r_0 \cos \theta \cos \theta$$

$v = \frac{v_0}{\cos^2 \theta}$	$v = \frac{v_0 r_0^2}{r^2}$
---------------------------------	-----------------------------

12.79-

$$F_m = m a_n$$

$$F_m = \frac{GMm}{r^2}$$

$$\frac{GMm}{r^2} = m \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

T - período

$$GM = g R_T^2$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{g R_T^2 \cdot T^2}{4\pi^2}$$

$$T = 23 \text{ h } 56 \text{ min} = 86.160 \text{ s}$$

$$g = 9,81 \text{ m/s}^2$$

$$R_T = 6,37 \times 10^6 \text{ m}$$

$$r^3 = \frac{9,81 \times (6,37 \times 10^6)^2 \times (86.160)^2}{4 \times \pi^2}$$

$$r = 42,14 \times 10^6 \text{ m}$$

a) altitude d:

$$d = r - R$$

$$d = 42,14 \times 10^6 - 6,37 \times 10^6$$

$$d = 35,77 \times 10^6 \text{ m}$$

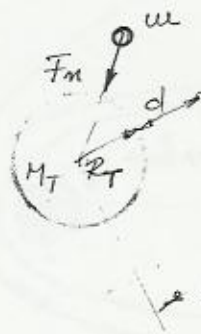
$$d = 35.774 \text{ km}$$

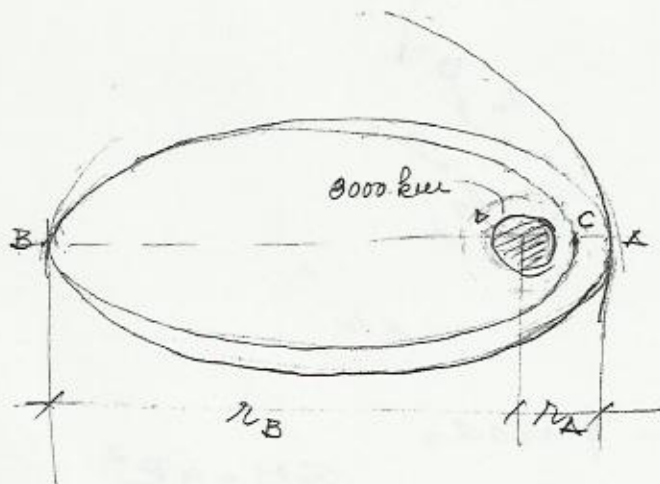
$$b) v = \frac{2\pi r}{T}$$

$$v = \frac{2\pi \times 42,14 \times 10^6}{86.160}$$

$$v = 3.073 \text{ m/s}$$

$$v = 11.063 \text{ km/h}$$





$$r_C = 8000 \text{ km}$$

$$r_A = 12 \times 10^3 \text{ km}$$

$$r_B = 96 \times 10^3 \text{ km}$$

$$GM_T = g R_T^2 = 9,81 \frac{\text{m}}{\text{s}^2} \times (637 \times 10^6 \text{ m}) = 3,98 \times 10^{14} \frac{\text{m}^3}{\text{s}^2}$$

$$v_B = 869 \text{ m/s}$$

$$v_A = 7400 \text{ m/s}$$

$$\Delta v_B = -146 \text{ m/s}$$

1ª TRANSFERÊNCIA DE ÓRBITA
CONSERVAÇÃO DO MOMENTO ANGULAR:

$$v_A r_A = v_B r_B$$

$$v_A \times 12 \times 10^3 = 869 \times 96 \times 10^3$$

$$v_A = 6.952 \text{ m/s}$$

$$\Delta v_A = -448 \text{ m/s}$$

$$\Delta v_A = 448 \text{ m/s}$$

2ª TRANSFERÊNCIA DE ÓRBITA

$$v_B = 869 - 146 = 723 \text{ m/s}$$

VELOCIDADE DE C EM TORNO DA
ÓRBITA CIRCULAR.

$$v_C = \sqrt{\frac{GM_M}{r_C}}$$

$$v_C = \sqrt{\frac{3,26 \times 10^{14}}{8 \times 10^6}}$$

$$v_C = 6.383,57 \text{ m/s}$$

$$GM_M = 0,82 GM_T$$

$$GM_M = 0,82 \times 3,98 \times 10^{14} \text{ m}^3/\text{s}^2$$

$$GM_M = 3,26 \times 10^{14} \text{ m}^3/\text{s}^2$$

$$\Delta v_C = -2.292,43 \text{ m/s}$$

$$r_C v_C = r_B v_B$$

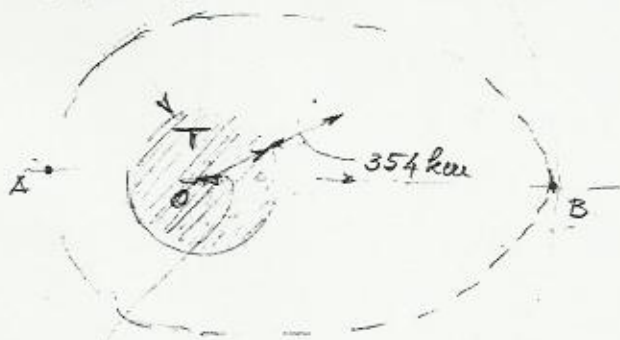
$$8 \times 10^6 v_C = 96 \times 10^3 \times 723$$

$$v_C = 8.676 \text{ m/s}$$

$$\Delta v_C = 2.292,43 \text{ m/s}$$

12.84

$$35,8 \times 10^3 \text{ km}$$



$$R = 6,37 \times 10^3 \text{ km}$$

$$r = (6,37 \times 10^3 + 354) \text{ km}$$

$$F = \cancel{m} a_n = \frac{GM \cancel{m}}{r^2}$$

$$\frac{v^2}{r} = \frac{GM}{r^2} = \frac{g R_T}{r}$$

VELOCIDADE CIRCULAR DE A:

$$v_A^2 = \frac{9,81 \times (6,37 \times 10^6)^2}{(6,37 \times 10^6 + 354 \times 10^3)} \quad v_A = 7.694,14 \text{ m/s}$$

VELOCIDADE ELÍPTICA DE A

$$(v_A)_{el} = 2,41 \text{ km/s} \quad (v_A)_{el} = 2410 \text{ m/s}$$

VELOCIDADE ELÍPTICA DE B:

CONSERVAÇÃO DO MOMENTO ANGULAR.

$$r_A \cdot v_A = r_B \cdot v_B$$

$$(6,37 \times 10^6 + 354 \times 10^3) \times 2410 = (6,37 \times 10^6 + 35,8 \times 10^6) \cdot (v_B)_{el}$$

$$(v_B)_{el} = 384,27 \text{ m/s}$$

VELOCIDADE CIRCULAR DE B.

$$v_B^2 = \frac{9,81 \times (6,37 \times 10^6)^2}{(6,37 \times 10^6 + 35,8 \times 10^6)} \quad v_B = 3.072 \text{ m/s}$$

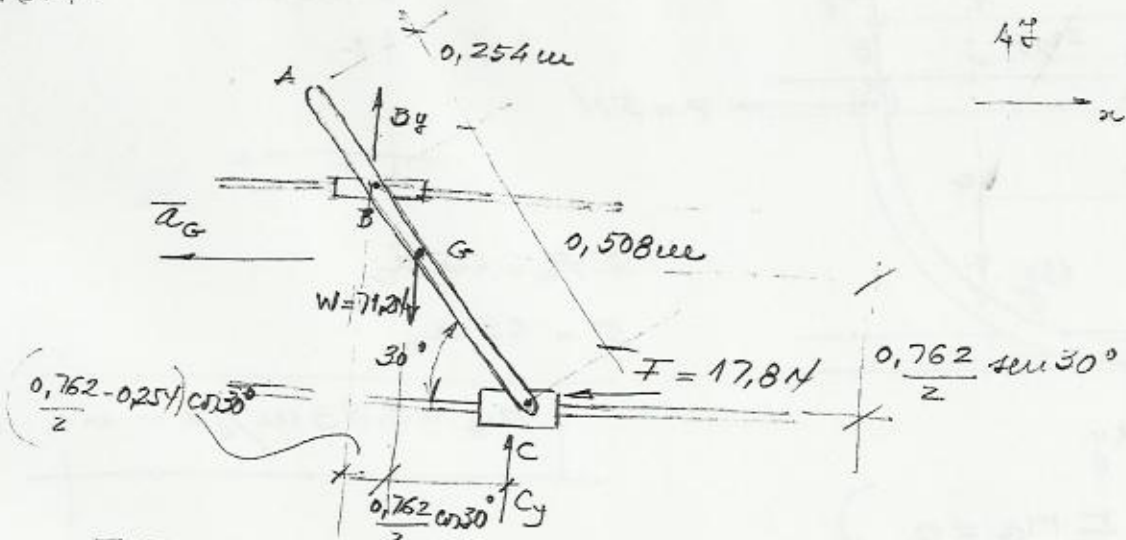
$$\Delta v_B = 3.072 - 384,27$$

$$\Delta v_B = 2.688 \text{ m/s}$$

$$\Delta v_B = 2,69 \text{ km/s}$$

Cap. 15

16.1-



$$\sum F_x = m \bar{a}_G$$

$$17,8 = \frac{71,2}{g} \times a_G$$

$$\boxed{a_G = 2,45 \text{ m/s}^2 \leftarrow}$$

$$\sum M_G = 0$$

$$C_y \times \frac{0,762}{2} \cos 30^\circ - B_y \left(\frac{0,762}{2} - 0,254 \right) \cos 30^\circ - F \times \frac{0,762}{2} \sin 30^\circ = 0$$

$$0,33 C_y - 0,11 B_y - 3,39 = 0$$

$$C_y = 10,28 + 0,33 B_y$$

$$\sum F_y = 0$$

$$C_y + B_y - 71,2 = 0$$

$$\begin{aligned} 10,28 + 0,33 B_y + B_y &= 71,2 \\ \boxed{B_y = 45,69 \text{ N}} & \quad \boxed{C_y = 25,51 \text{ N}} \end{aligned}$$

16.2-

$$B_y = 35,6 \text{ N} \uparrow$$

$$B_y + C_y = 71,2$$

$$C_y = 35,6 \text{ N} \uparrow$$

$$\sum M_G = 0$$

$$0,33 C_y - 0,11 B_y - 0,19 F = 0$$

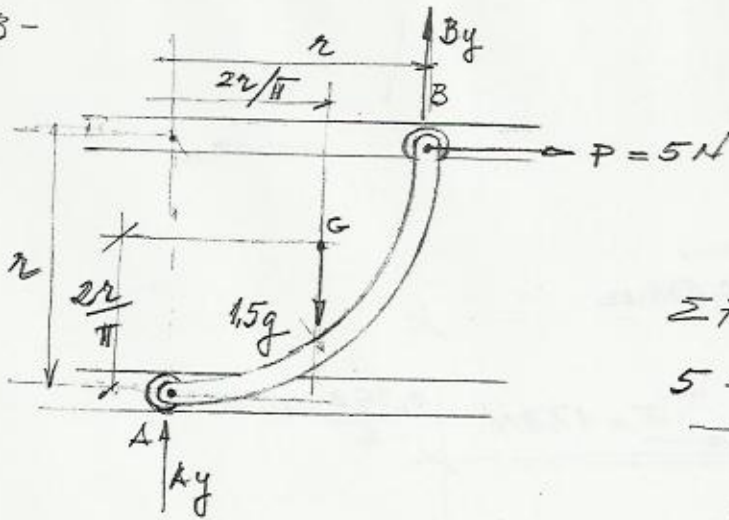
$$\boxed{F = 41,11 \text{ N} \leftarrow}$$

$$\sum F_x = m \bar{a}_G$$

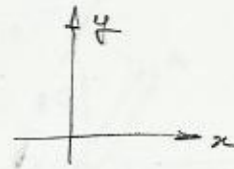
$$41,11 = \frac{71,2}{g} \cdot \bar{a}_G$$

$$\boxed{\bar{a}_G = 5,66 \text{ m/s}^2 \leftarrow}$$

16.3-



$r = 0,3m$



$\sum F_x = m \bar{a}_G$

$5 = 1,5 a_G$

$a_G = 3,33 m/s^2 \rightarrow$

$\sum M_G = 0$

$A_y \times \frac{2r}{\pi} - B_y \left(r - \frac{2r}{\pi} \right) + P \left(r - \frac{2r}{\pi} \right) = 0$

$0,19 A_y - 0,81 B_y + 4,05 = 0$

$A_y = 4,23 B_y - 21,18$

$\sum F_y = 0$

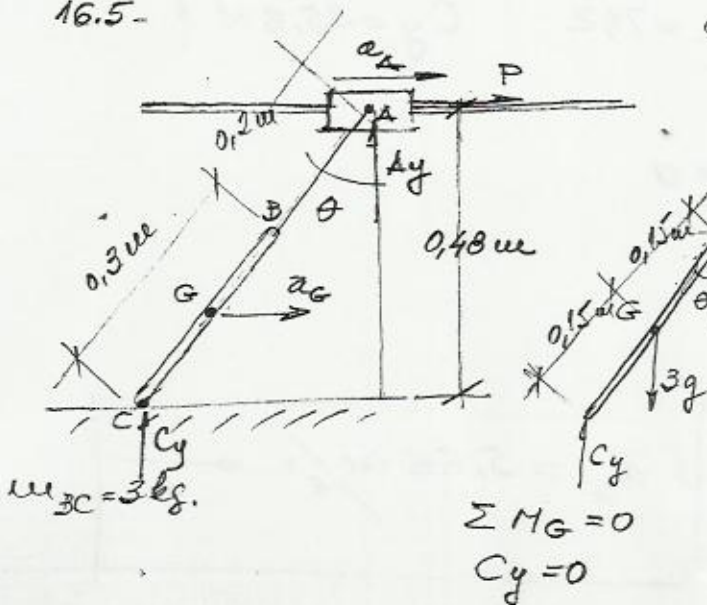
$A_y + B_y = 1,5g$

$4,23 B_y + B_y - 21,18 = 1,5g$

$B_y = 6,86 N$

$A_y = 7,86 N$

16.5-



CORDA E BARRA ALINHADAS:

$\cos \theta = \frac{0,48}{0,5}$

$\theta = 16,26^\circ$

$a_G = a_A$ (TRANSLAÇÃO)

$\sum F_x = m \bar{a}_G$

$T \cos \theta = 3 \cdot a_G$

$T = 10,71 a_G$

$a_G = 2,86 m/s^2$

$\sum F_y = 0$

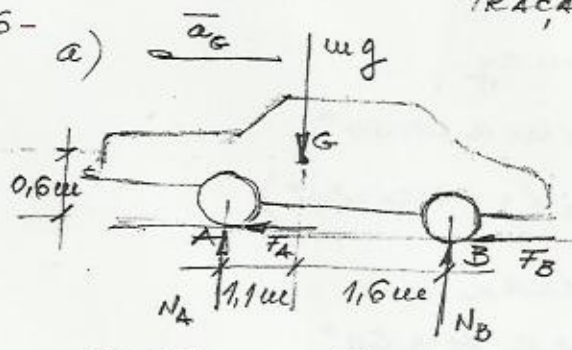
$T \sin \theta = 3g$

$T = 30,66 N$

$a_G = 2,86 m/s^2 \rightarrow$

16.6-

TRACÃO NAS QUATRO RODAS



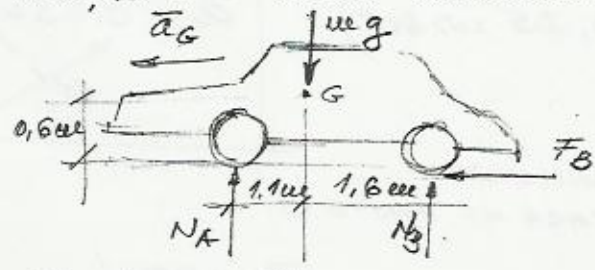
$\mu_e = 0,80$
 $F_A = \mu_e N_A$
 $F_B = \mu_e N_B$
 $F_A + F_B = \mu_e (N_A + N_B)$
 $\sum F_y = 0$
 $N_A + N_B = mg$

$\sum F_x = m \bar{a}_G$
 $F_A + F_B = m \bar{a}$
 $\mu_e (N_A + N_B) = m \bar{a}$
 $\mu_e (mg) = \mu_e \bar{a}$

$\bar{a}_G = \mu_e g$

$\bar{a}_G = 7,85 \text{ m/s}^2$

b) TRACÃO NAS RODAS TRASEIRAS.



$\mu_e = 0,80$
 $F_B = \mu_e N_B$
 $\sum F_y = 0 \quad N_A + N_B = mg$

$\sum M_G = 0 \quad (+)$
 $N_A \times 1,1 + F_B \times 0,6 - N_B \times 1,6 = 0$
 $N_A \times 1,1 + \mu_e N_B \times 0,6 - 1,6 N_B = 0$
 $N_A = 1,02 N_B$

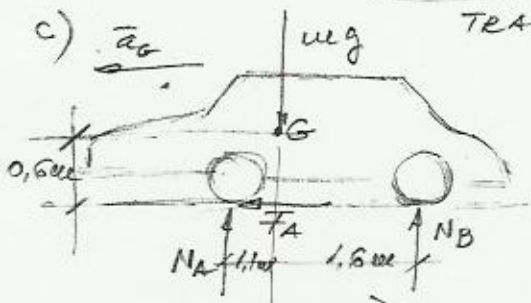
$1,02 N_B + N_B = mg$
 $N_B = \frac{mg}{2,02}$

$F_B = \frac{0,8}{2,02} mg$

$\sum F_x = m \bar{a}_G$
 $\frac{0,8}{2,02} mg = \mu_e \bar{a}_G$

$\bar{a}_G = 3,89 \text{ m/s}^2$

TRACÃO NAS RODAS DIANTEIRAS.



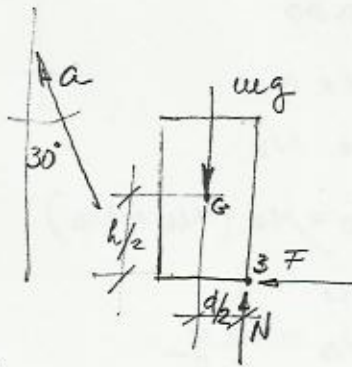
$F_A = \mu_e N_A = 0,8 N_A$
 $\sum F_y = 0 \quad N_A + N_B = mg$
 $N_A + 0,98 N_A = mg$
 $N_A = 0,51 mg$
 $F_A = 0,8 \times 0,51 mg = 0,404 mg$

$\sum M_G = 0 \quad (+)$
 $N_A \times 1,1 + F_A \times 0,6 - N_B \times 1,6 = 0$
 $N_A = 1,02 N_B$
 $N_B = 0,98 N_A$

$\sum F_x = m \bar{a}_G$
 $F_A = \mu_e \bar{a}_G$
 $0,404 mg = \mu_e \bar{a}_G$

$\bar{a}_G = 3,96 \text{ m/s}^2$

16.10 -



$$\Sigma F_y = ma_y$$

$$N - mg = ma \cos 30^\circ$$

$$N = m(g + a \cos 30^\circ)$$

$$\Sigma F_x = ma_x$$

$$F = ma \sin 30^\circ$$

a)

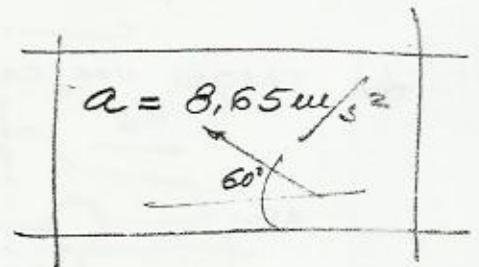
LATAS DESLIZANDO

$$F = \mu_c N$$

$$\mu_c = \frac{\mu a \sin 30^\circ}{\mu(g + a \cos 30^\circ)}$$

$$0,25(g + a \cos 30^\circ) = a \sin 30^\circ$$

$$0,25g = a(\sin 30^\circ - 0,25 \cos 30^\circ)$$



b) NA SITUAÇÃO DE TOMBAMENTO A NORMAL ESTÁ APLICADA NO PONTO B.

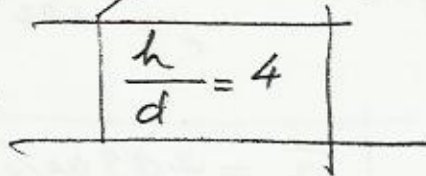
$$\Sigma M_G = 0 \quad (\oplus)$$

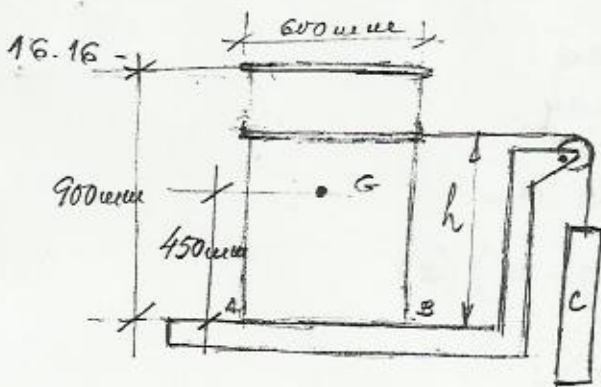
$$F = 0,25 N$$

$$N \times \frac{d}{2} = F \frac{h}{2}$$

$$\frac{N}{F} = \frac{h}{d}$$

$$\frac{N}{0,25N} = \frac{h}{d}$$





$$m_b = 125 \text{ kg}$$

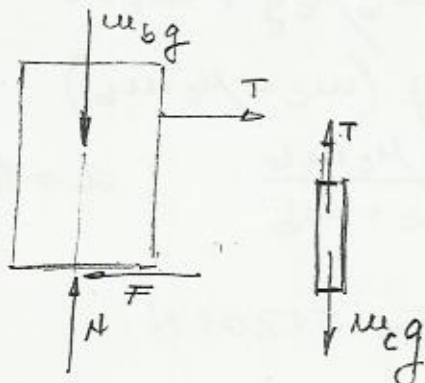
$$m_c = 60 \text{ kg}$$

$$\mu_e = 0,35$$

$$\mu_c = 0,30$$

1) VERIFICAÇÃO DO ESTADO DE DESLIZAMENTO.

ASSUMINDO O REPOUSO (IMINÊNCIA DE MOVIMENTO) E DETERMINAMOS O VALOR MÍNIMO DE μ_c P/ ESTA SITUAÇÃO.



P/ O BARRIL

$$\sum F_x = 0 \quad F = T$$

$$\sum F_y = 0 \quad N = m_b g$$

$$F = \mu_e N$$

$$F = \mu_e m_b g = T$$

P/ O CILINDRO $\sum F_y = 0$

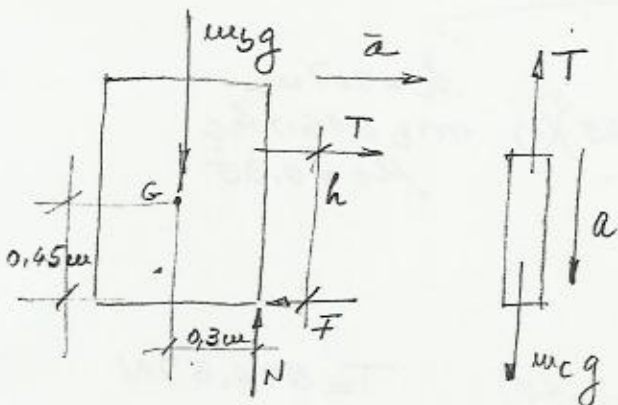
$$T = m_c g$$

$$\mu_e m_b g = m_c g \quad \mu_c = \mu_e m_b$$

$$m_c = 0,35 \times 125 \quad m_c = 43,75 \text{ kg}$$

COMO $m_c = 60 \text{ kg} > 43,75 \text{ kg}$ — O CORPO ESTÁ DESLIZANDO.

ASSUMINDO O DESLIZAMENTO E A IMINÊNCIA DE TOMBAMENTO.



P/ O BARRIL

$$F = \mu_c N \quad \sum F_y = 0 \quad N = m_b g$$

$$F = \mu_c m_b g$$

$$\sum F_x = m_b a$$

$$T - F = m_b a$$

$$T - \mu_c m_b g = m_b a$$

$$T = m_b (\mu_c g + a) \quad (1)$$

$$\sum M_G = 0 \quad (+)$$

$$T(h - 0,45) + F \times 0,45 - N \times 0,3 = 0$$

$$T(h - 0,45) = m_b g (0,3 - \mu_c \times 0,45) \quad (2)$$



P/O CILINDRO
 $\Sigma F_y = m a_y$

$$m_c g - T = m_c \cdot a$$

$$T = m_c (g - a) \quad (3)$$

Subst. (3) em (1):

$$m_c (g - a) = m_b (\mu_c g + a)$$

$$m_c g - m_c a = m_b \mu_c g + m_b a$$

$$a (m_b + m_c) = g (m_c - \mu_c m_b)$$

$$a = g \frac{m_c - \mu_c m_b}{m_c + m_b} \quad a = 1,19 \text{ m/s}^2 \rightarrow$$

$$T = m_c (g - a)$$

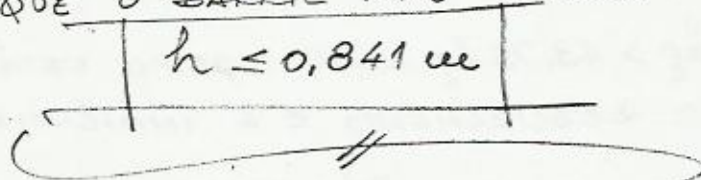
$$T = 60 (9,81 - 1,19)$$

$$T = 517,01 \text{ N}$$

Subst o valor de T em (2)

$$517,01 (h - 0,45) = 125 \times 9,81 (0,3 - 0,3 \times 0,45) \quad h = 0,841 \text{ m}$$

P/ QUE O BARRIL NÃO TOMBE.



16.17 - DO PROBLEMA ANTERIOR:

$$h = 0,7 \text{ m}$$

$$T (h - 0,45) = m_b g (0,3 - \mu_c \times 0,45) \quad (1) \quad m_b = 160 \text{ kg}$$

$$\mu_c = 0,35$$

$$T = m_c (g - a) \quad (2)$$

$$T = m_b (\mu_c g + a) \quad (3)$$

Eq (1):

$$T (0,7 - 0,45) = 160 \times 9,81 (0,3 - 0,35 \times 0,45)$$

$$T = 894,67 \text{ N}$$

Subst. T em eq. (3).

$$894,67 = 160 (0,35 \times 9,81 + a)$$

$$a = 2,16 \text{ m/s}^2.$$

Subst. a e T em eq (2).

$$894,67 = m_c (9,81 - 2,16)$$

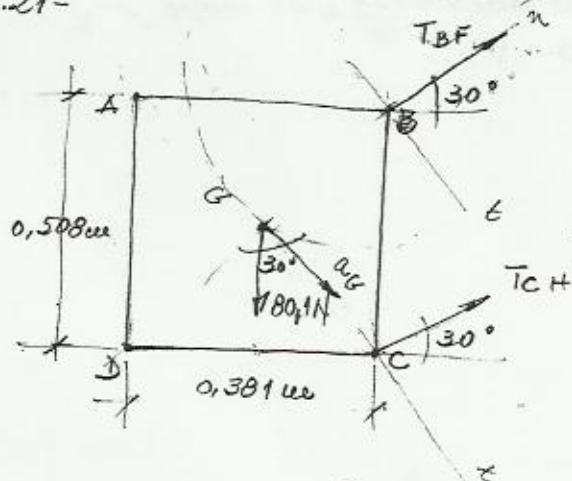
$$m_c = 116,95 \text{ kg.}$$

VALOR MÁXIMO DE m_c .
 (IMINÊNCIA DE TOMBAMENTO)

$$m_c \leq 116,95 \text{ kg}$$

16.21-

IMEDIATAMENTE APÓS O CABO AE TER SIDO CORTADO.



$$\Sigma F_x = m a_x$$

$$80,1 \cos 30^\circ = \frac{80,1}{g} \times a_G$$

$$a_G = g \cos 30^\circ$$

a)

$$a_G = 8,50 \text{ m/s}^2$$

30°

$$b) \Sigma F_n = 0$$

$$T_{BF} + T_{CH} - 80,1 \sin 30^\circ = 0$$

$$T_{BF} + T_{CH} = 40,05 \text{ N} \quad (1)$$

$$\Sigma M_G = 0 \quad (+)$$

$$T_{BF} \cos 30^\circ \times \frac{0,508}{2} - T_{BF} \sin 30^\circ \times \frac{0,381}{2} - T_{CH} \cos 30^\circ \times \frac{0,508}{2} - T_{CH} \sin 30^\circ \times \frac{0,381}{2} = 0$$

$$0,125 T_{BF} = 0,315 T_{CH}$$

$$T_{BF} = 2,527 T_{CH} \quad (2)$$

Subst. (2) em (1):

$$2,527 T_{CH} + T_{CH} = 40,05$$

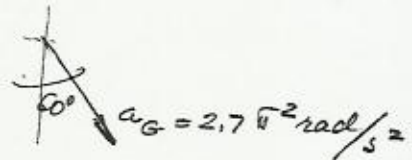
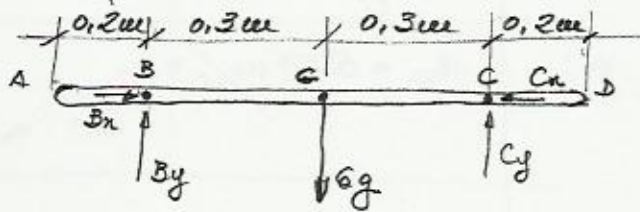
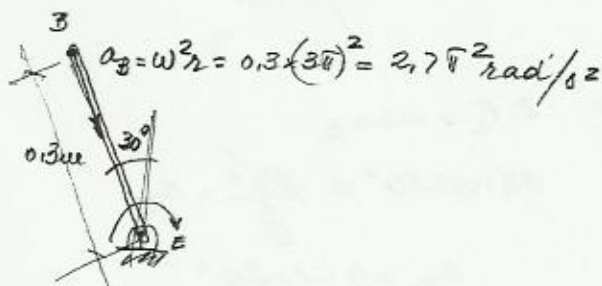
$T_{CH} = 11,35 \text{ N}$
$T_{BF} = 28,69 \text{ N}$

16.23-

$$\omega_{BE} = \omega_{CF} = 90 \text{ rpm} = \frac{90 \times \pi}{30} = 3\pi \text{ rad/s}$$

$$\alpha = 0$$

$$\bar{a}_G = a_B = a_C$$



$$\sum M_G = 0$$

$$B_y = C_y$$

$$\sum F_y = m a_{Gy}$$

$$B_y - 6g + C_y = -6 \cdot 2.7\pi^2 \cos 60^\circ$$

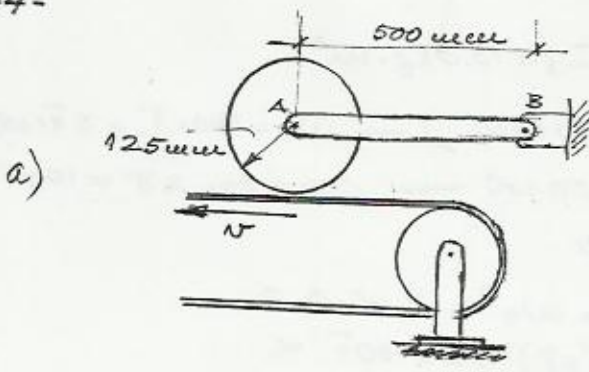
$$B_y + C_y = -21.08$$

$$2B_y = -21.08$$

$$B_y = -10.54 \text{ N}$$

$B_y = 10.54 \text{ N} \downarrow$
$C_y = 10.54 \text{ N} \downarrow$

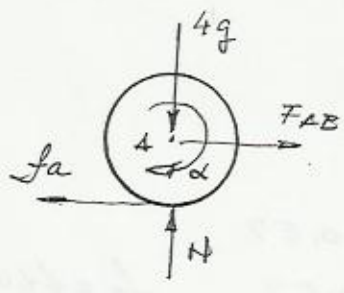
16.34-



$$\bar{I}_A = \frac{1}{2} m r^2$$

$$m = 4 \text{ kg}$$

$$\mu_c = 0,40$$



$$\sum F_y = 0$$

$$N = 4g$$

$$f_a = \mu_c N$$

$$f_a = 0,4 \times 4g$$

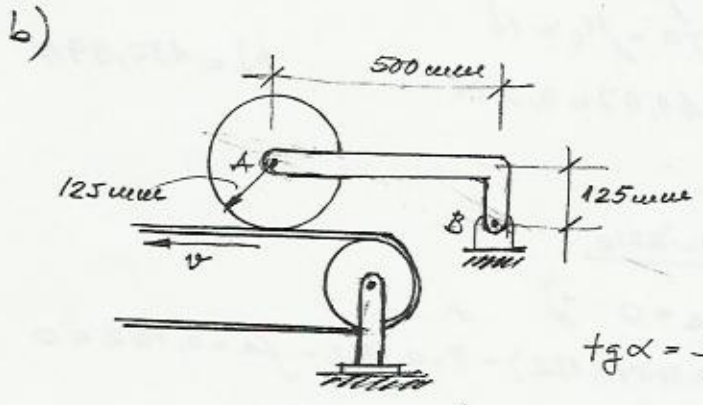
$$f_a = 1,6g$$

$$\sum M_A = \bar{I}_A \alpha$$

$$f_a \times r = \frac{m r^2}{2} \alpha$$

$$1,6g = \frac{4 \times 0,125}{2} \alpha$$

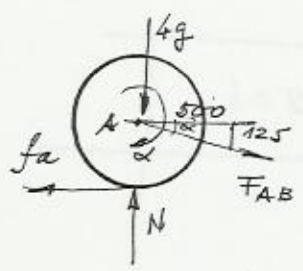
$$\alpha = 62,78 \text{ rad/s}^2$$



$$\tan \alpha = \frac{125}{500}$$

$$\cos \alpha = 0,97$$

$$\sin \alpha = 0,24$$



$$\sum F_x = 0$$

$$F_{AB} \cos \alpha = f_a \quad (1)$$

$$\sum F_y = 0$$

$$N - F_{AB} \sin \alpha - 4g = 0$$

$$F_{AB} \sin \alpha = N - 4g \quad (2)$$

Dividendo eq(2)/eq(1) tenemos:

$$\tan \alpha = \frac{N - 4g}{f_a} \quad f_a = 17,44 \text{ N}$$

$$f_a = \mu_c N$$

$$0,25 = \frac{N - 4g}{0,4N}$$

$$N = 43,60 \text{ N}$$

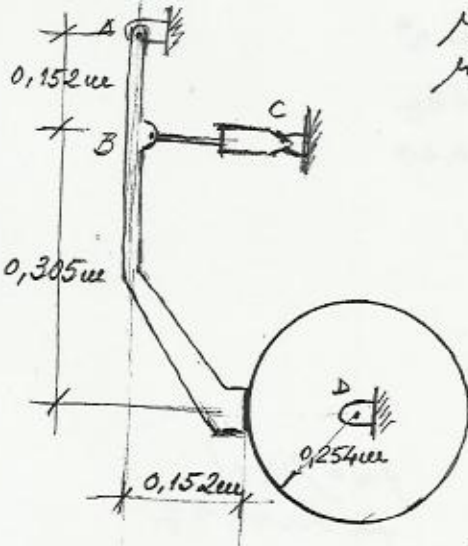
$$\sum M_A = \bar{I}_A \alpha$$

$$f_a \times r = \frac{m r^2}{2} \alpha$$

$$\frac{17,44 \times 2}{4 \times 0,125} = \alpha$$

$$\alpha = 69,76 \text{ rad/s}^2$$

16.38-



$\mu_e = 0,40$
 $\mu_c = 0,30$

$I_D = 18,3 \text{ kg} \cdot \text{m}^2$

$\omega_0 = 180 \text{ rpm} \rightarrow \omega_0 = 180 \frac{2\pi}{60} = 6\sqrt{\pi} \text{ rad/s}$
 $\Delta\theta = 50 \text{ rev} \rightarrow \Delta\theta = 50 \times 2\pi = 100\pi \text{ rad}$

$\omega_f = 0$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$0 = (6\sqrt{\pi})^2 + 2 \times 100\pi \cdot \alpha$

$-\frac{36\pi}{200} = \alpha \quad \alpha = 0,57 \text{ rad/s}^2$

$\sum M_D = I_D \alpha$

$f_a \times r = 18,3 \times 0,57$

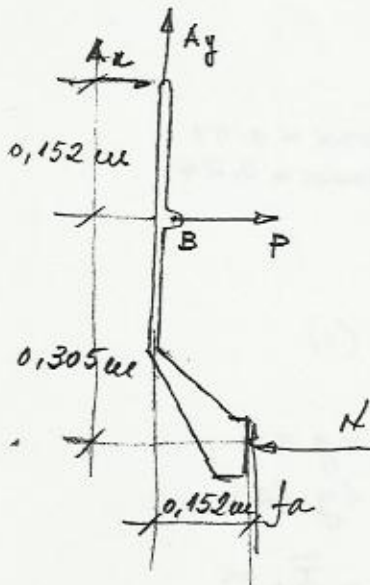
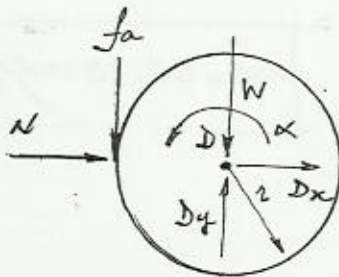
$f_a = \frac{18,3 \times 0,57}{0,254}$

$f_a = 41,07 \text{ N}$

$f_a = \mu_c \times N$

$41,07 = 0,3 N$

$N = 136,89 \text{ N}$



EQUILIBRIO

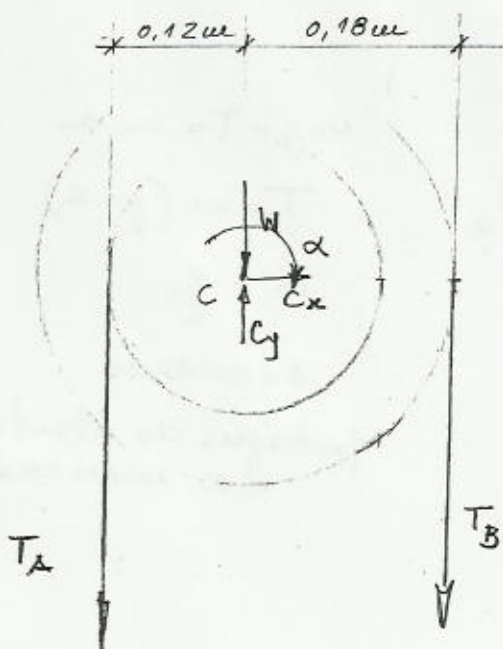
$\sum M_A = 0$

$N \times (0,305 + 0,152) - P \times 0,152 - f_a \times 0,152 = 0$

$\frac{136,89 \times 0,457 - 41,07 \times 0,152}{0,152} = P$

$P = 368 \text{ N}$

16.42 -



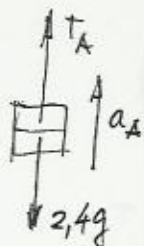
$$a_A = 0,12 \alpha$$

$$a_B = 0,18 \alpha$$

$$\Sigma F = m a$$

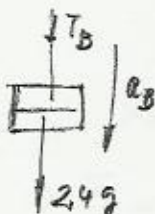
$$T_A - 2,4g = 2,4 a_A$$

$$T_A = 2,4(g + 0,12 \alpha)$$



$$2,4g - T_B = 2,4 a_B$$

$$T_B = 2,4(g - 0,18 \alpha)$$



$$I_c = m k_c^2$$

$$I_c = 6 \times 0,135^2$$

$$I_c = 0,109 \text{ kg m}^2$$

$$a) \quad \Sigma M_c = \bar{I}_c \alpha$$

$$T_B \times 0,18 - T_A \times 0,12 = 0,1094 \alpha$$

$$2,4(g - 0,18 \alpha) \times 0,18 - 2,4(g + 0,12 \alpha) \times 0,12 = 0,1094 \alpha$$

$$2,4g(0,18 - 0,12) - 2,4(0,18^2 + 0,12^2) \alpha = 0,1094 \alpha$$

$$\boxed{\alpha = 6,37 \text{ rad/s}^2}$$

$$b) \quad a_A = 0,12 \alpha$$

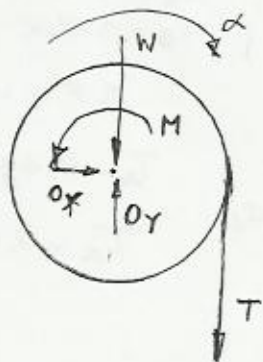
$$a_A = 0,76 \text{ m/s}^2 \uparrow$$

$$v_A = v_0 + a_A t$$

$$v_A = 0,76 \times 2,5$$

$$\boxed{v_A = 1,91 \text{ m/s} \uparrow}$$

16.44-



$$mg - T = ma$$

$$T = m(g - a)$$

$$\alpha = \frac{a}{r}$$

$$r = 0,610 \text{ m}$$

$M \rightarrow$ torque do atrito nos mancais.

$$\sum M_o = \bar{I} \alpha$$

$$T \times r - M = \bar{I} \alpha$$

$$m(g - a) \times r - M = \bar{I} \alpha$$

$$m(g - a) \times r - M = \bar{I} \cdot \frac{a}{r}$$

1º caso: $m_1 = \frac{89 \text{ N}}{g}$ $\Delta y = 3,05 \text{ m}$ $t = 4,6 \text{ s}$.

$$\Delta y = \frac{1}{2} a t^2 \quad a_1 = 0,29 \text{ m/s}^2$$

2º caso: $m_2 = \frac{178 \text{ N}}{g}$ $\Delta y = 3,05 \text{ m}$ $t = 3,1 \text{ s}$

$$\Delta y = \frac{1}{2} a t^2 \quad a_2 = 0,63 \text{ m/s}^2$$

P/ A PRIMEIRA SITUAÇÃO:

$$\frac{89}{g} (g - 0,29) \times 0,610 - M = \bar{I} \frac{0,29}{0,610} \Rightarrow 52,69 - M = 0,475 \bar{I} \quad (1)$$

P/ A SEGUNDA SITUAÇÃO:

$$\frac{178}{g} (g - 0,63) \times 0,610 - M = \bar{I} \frac{0,63}{0,610} \Rightarrow 101,61 - M = 1,033 \bar{I} \quad (2)$$

FAZENDO eq (2) - eq (1):

$$101,61 - 52,69 = (1,033 - 0,475) \bar{I}$$

$$\boxed{\bar{I} = 87,67 \text{ kg} \cdot \text{m}^2}$$

16.45. DO PROBLEMA 16.44.

$$m(g-a)r - M = \bar{I} \frac{a}{r}$$

DESPREZANDO O ATRITO $\Rightarrow M = 0$

$$\bar{I} = m_v \bar{k}^2$$

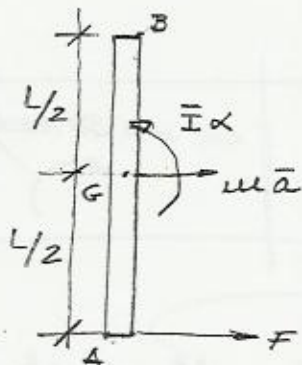
$$m(g-a)r = m_v \bar{k}^2 \frac{a}{r}$$

$$mgr - mar = m_v \bar{k}^2 \frac{a}{r}$$

$$mgr^2 = (m_v \bar{k}^2 + mr^2) a$$

$$a = \frac{mgr^2}{(m_v \bar{k}^2 + mr^2)}$$

16.59.



$$L = 900 \text{ mm.}$$

$$I_G = \frac{mL^2}{12}$$

$$\Sigma F_x = ma_x$$

$$F = ma$$

$$3 = 1,25 a$$

$$m = 1,25 \text{ kg}$$

$$L = 900 \text{ mm}$$

$$F = 3 \text{ N}$$

$$a = 2,4 \text{ m/s}^2 \rightarrow$$

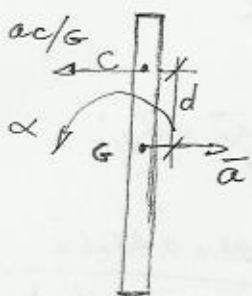
$$\Sigma M_G = \bar{I} \alpha$$

$$F \times \frac{L}{2} = \frac{mL^2}{12} \alpha$$

$$3 \times 0,45 = \frac{1,25 \times 0,9^2}{12} \alpha$$

$$\alpha = 16 \text{ rad/s}^2$$

PONTO C DE ACELERAÇÃO NULA



$$\vec{a}_C = \vec{a}_G + \vec{a}_{c/G}$$

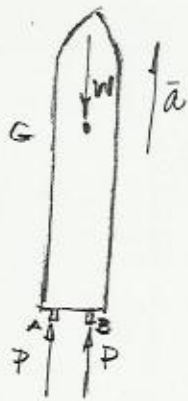
$$a_c = a - \alpha d$$

$$a_c = 0$$

$$0 = 2,4 - 16 \times d$$

$$d = 0,15 \text{ m}$$

16.61- COM OS DOIS MOTORES FUNCIONANDO



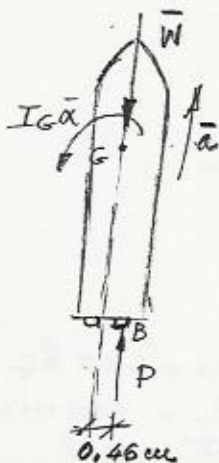
$$2P - W = m \bar{a} \quad W = 1,11 \times 10^5 \text{ N} \\ \bar{a} = 13,7 \text{ m/s}^2$$

$$2P - 1,11 \times 10^5 = \frac{1,11 \times 10^5}{9,81} \times 13,7$$

$$2P = 266.015 \text{ N}$$

$$P = 1,33 \times 10^5 \text{ N}$$

COM SOMENTE O MOTOR B FUNCIONANDO.



$$P - W = m \bar{a}$$

$$1,33 \times 10^5 - 1,11 \times 10^5 = \frac{1,11 \times 10^5}{9,81} \bar{a}$$

$$\bar{a} = 1,95 \text{ m/s}^2 \uparrow$$

$$\bar{I}_G = \frac{m l^2}{12}$$

$$\bar{I}_G = \frac{1,11 \times 10^5 \times 14,6^2}{9,81 \times 12} = 2,01 \times 10^5 \text{ kg m}^2$$

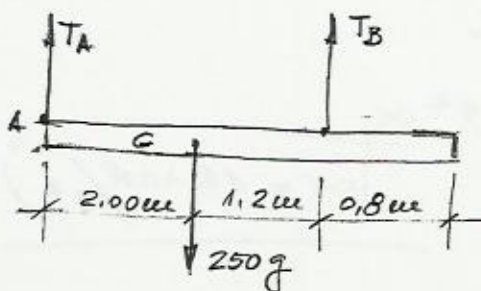
$$\sum M_G = I_G \alpha$$

$$P \times 0,46 = \bar{I}_G \alpha$$

$$1,33 \times 10^5 \times 0,46 = 2,01 \times 10^5 \alpha$$

$$\alpha = 0,30 \text{ rad/s}^2$$

16.62-



$$T_A = 1000 \text{ N} \\ T_B = 1800 \text{ N}$$

$$\bar{I} = \frac{m l^2}{12} = \frac{250 \times 4^2}{12} = 333,33 \text{ kg m}^2$$

$$\sum F_y = m a_y$$

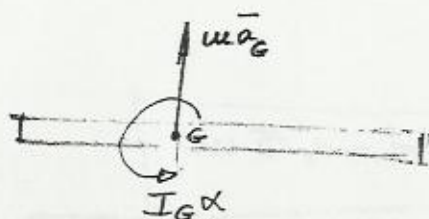
$$T_B + T_A - 250g = 250 \cdot \bar{a}_G$$

$$\bar{a}_G = 1,39 \text{ m/s}^2 \uparrow$$

$$\sum M_G = I_G \alpha$$

$$T_B \times 1,2 - T_A \times 2,00 = \bar{I} \alpha$$

$$\alpha = 0,48 \text{ rad/s}^2 \downarrow$$



$$\bar{a}_A = \bar{a}_G + a_{A/G}$$

$$a_A = 1,39 - 0,48 \times 2$$

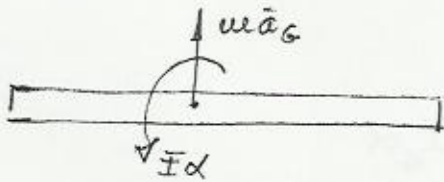
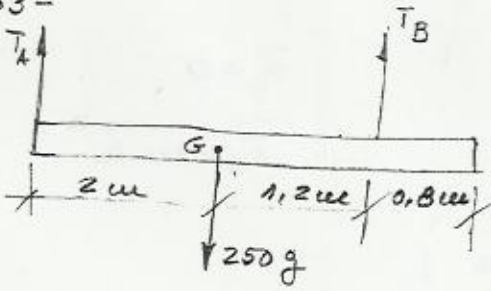
$$a_A = 0,43 \text{ m/s}^2 \uparrow$$

$$\bar{a}_B = \bar{a}_G + \bar{a}_{B/G}$$

$$a_B = 1,39 + 0,48 \times 1,2$$

$$a_B = 1,97 \text{ m/s}^2 \uparrow$$

16.63-



$$\bar{I} = 333,33 \text{ kg} \cdot \text{m}^2$$

$$a_B = 0,5 \text{ m/s}^2 \uparrow$$

$$a_A = 2 \text{ m/s}^2 \uparrow$$

$$\vec{a}_A = \vec{a}_G + \vec{a}_A/G$$

$$\vec{a}_B = \vec{a}_G + \vec{a}_B/G$$

$$2 = a_G - 2\alpha \quad (1)$$

$$0,5 = a_G + 1,2\alpha \quad (2)$$

Fazendo eq (1) - eq (2):

$$2 - 0,5 = -2\alpha - 1,2\alpha \quad \alpha = -0,47$$

$$\alpha = 0,47 \text{ rad/s}^2$$

$$2 = a_G - 2\alpha$$

$$2 = a_G - (-0,47) \times 2$$

$$a_G = 1,06 \text{ m/s}^2 \uparrow$$

$$\sum F_y = m a_y$$

$$T_A + T_B - 250g = 250 \cdot 1,06$$

$$T_A + T_B = 2.717,5 \text{ N} \quad (3)$$

Subst (3) em (4):

$$2(2.717,5 - T_B) - 1,2T_B = 156,67$$

$$-3,2T_B = -5.278,33$$

$$\sum M_G = I_G \alpha$$

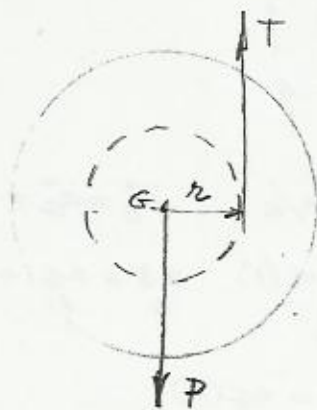
$$2T_A - 1,2T_B = 333,33 \times 0,47$$

$$2T_A - 1,2T_B = 156,67 \quad (4)$$

$$T_B = 1.649,48 \text{ N}$$

$$T_A = 1.068,02 \text{ N}$$

16.66.



$$\sum F_y = m a_y$$

$$T - P = 0$$

$$\boxed{T = P}$$



$$\sum M_G = I_G \alpha$$

$$T \cdot r = m k^2 \cdot \alpha$$

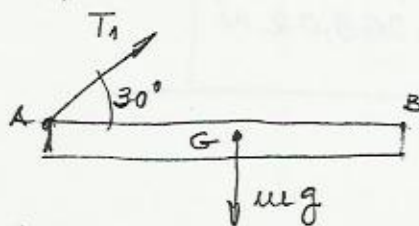
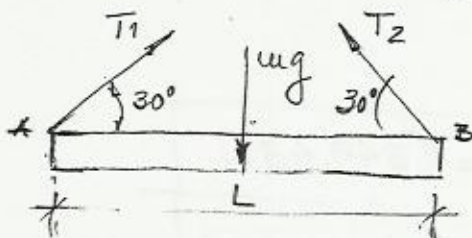
$$P \cdot r = \frac{P}{g} k^2 \cdot \alpha$$

$$\boxed{\alpha = \frac{g r}{k^2}}$$

$$\bar{a}_G = 0$$

$$I_G = m k^2$$

16.71



$$I_G = \frac{m L^2}{12}$$

$$\sum F_y = 0$$

$$\sum F_x = 0$$

$$T_1 \sin 30^\circ + T_2 \sin 30^\circ = m g$$

$$2 T_1 \sin 30^\circ = m g$$

$$T_1 = m g$$

$$\sum F_x = m \bar{a}_x$$

$$T_1 \cos 30^\circ = m \bar{a}_x$$

$$m g \frac{\sqrt{3}}{2} = m \bar{a}_x$$

$$\bar{a}_x = \frac{\sqrt{3}}{2} g \rightarrow$$

$$\sum F_y = m \bar{a}_y$$

$$m g - m g \sin 30^\circ = m \bar{a}_y$$

$$m g - \frac{m g}{2} = m \bar{a}_y$$

$$\bar{a}_y = \frac{g}{2} \downarrow$$

$$\sum M_G = I_G \alpha$$

$$T_1 \sin 30^\circ \times \frac{L}{2} = \frac{m L^2}{12} \alpha$$

$$m g \times \frac{1}{2} \times \frac{L}{2} = \frac{m L^2}{12} \alpha$$

$$\boxed{\alpha = \frac{3g}{L}}$$

16.71 (cont.)

$$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$$

$$\vec{a}_A = \left(\frac{\sqrt{3}}{2}g \rightarrow + \frac{g}{2} \downarrow \right) + \alpha \frac{L}{2} \uparrow$$

Comp \rightarrow :

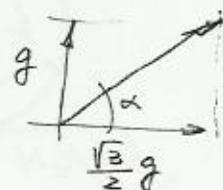
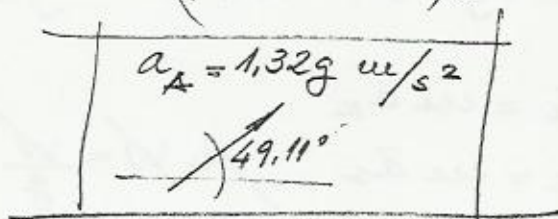
$$(a_A)_x = \frac{\sqrt{3}}{2}g \rightarrow$$

Comp \uparrow

$$(a_A)_y = -\frac{g}{2} + \frac{3g}{2} \cdot \frac{L}{2}$$

$$(a_A)_y = g \uparrow$$

$$a_A = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + 1^2} g$$



$$\tan \alpha = \frac{1}{\frac{\sqrt{3}}{2}}$$

$$\alpha = 49.11^\circ$$

$$\vec{a}_B = \vec{a}_G + \vec{a}_{B/G}$$

$$\vec{a}_B = \left(\frac{\sqrt{3}}{2}g \rightarrow + \frac{g}{2} \downarrow \right) + \alpha \frac{L}{2} \downarrow$$

Comp \rightarrow :

$$(a_B)_x = \frac{\sqrt{3}}{2}g \rightarrow$$

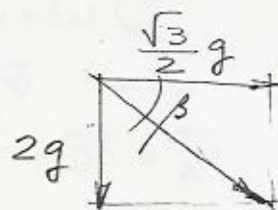
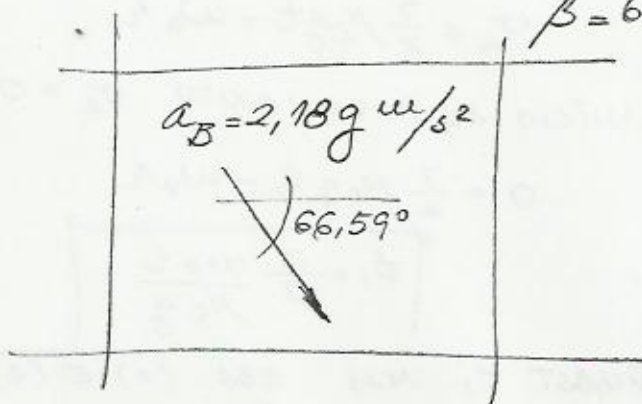
Comp \downarrow :

$$(a_B)_y = -\frac{g}{2} - \frac{3g}{2} \cdot \frac{L}{2}$$

$$(a_B)_y = -2g$$

$$(a_B)_y = 2g \downarrow$$

$$a_B = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + 2^2} g$$

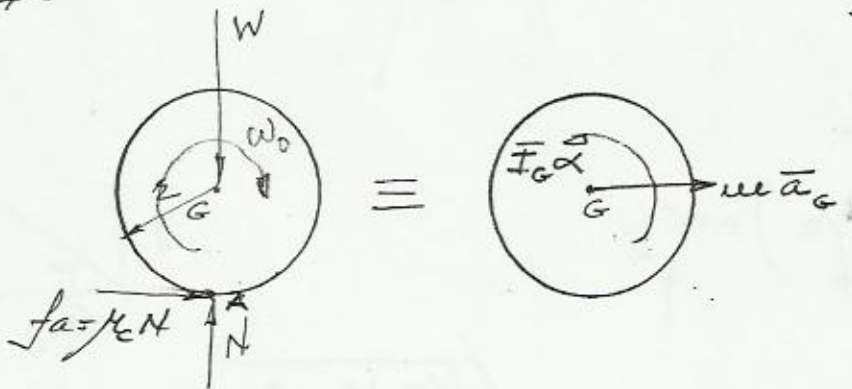


$$\tan \beta = \frac{2}{\frac{\sqrt{3}}{2}}$$

$$\beta = 66.59^\circ$$

16.74-

$$I_G = \frac{2}{5} m r^2$$



$$\sum F_y = 0 \quad N = W \quad f_a = \mu_c N \Rightarrow f_a = \mu_c W$$

$$\sum F_x = m a_x$$

$$f_a = m \bar{a}_G \quad \mu_c W = \frac{m}{g} \bar{a}_G$$

$$\bar{a}_G = \mu_c g \rightarrow$$

$$\sum M_G = I_G \alpha$$

$$f_a \times r = \frac{2}{5} m r^2 \alpha$$

$$\mu_c W = \frac{2}{5} \frac{m}{g} r \alpha$$

$$\alpha = \frac{5}{2} \frac{\mu_c g}{r}$$

CINEMÁTICA:

$$+) \quad \omega = \omega_0 - \alpha t = \omega_0 - \frac{5}{2} \mu_c \frac{g}{r} t \quad (1)$$

$$\rightarrow \quad \bar{v} = \bar{a} t = \mu_c g t = v_G \quad (2)$$

$$\vec{v}_A = \vec{v}_G + \vec{v}_{A/G}$$

$$v_A = \mu_c g t - \omega r$$

$$v_A = \mu_c g t - \left(\omega_0 - \frac{5}{2} \mu_c \frac{g}{r} \right) r$$

$$v_A = \frac{7}{2} \mu_c g t - \omega_0 r$$

a) INÍCIO DO ROLAMENTO $v_A = 0 \Rightarrow t = t_1$

$$0 = \frac{7}{2} \mu_c g t_1 - \omega_0 r$$

$$t_1 = \frac{2}{7} \frac{\omega_0 r}{\mu_c g}$$

b) SUBST. t_1 NAS EQS. (1) E (2) TEMOS:

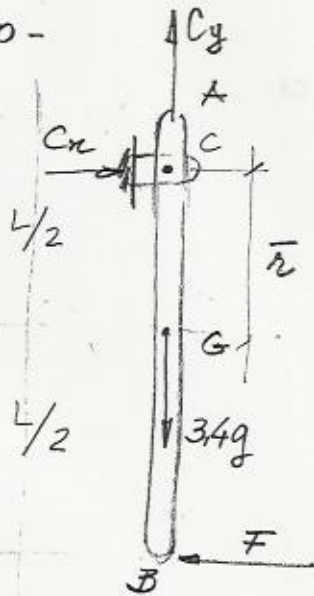
$$\omega = \omega_0 - \frac{5}{2} \mu_c \frac{g}{r} \times \frac{2}{7} \frac{\omega_0 r}{\mu_c g}$$

$$\omega = \frac{2}{7} \omega_0 \rightarrow$$

$$\bar{v} = \mu_c g \times \frac{2}{7} \frac{\omega_0 r}{\mu_c g}$$

$$\bar{v} = \frac{2}{7} \omega_0 r \rightarrow$$

16.80 -



$$\bar{r} = \frac{L}{4}$$

$$L = 0,914 \text{ m}$$

$$m = 3,4 \text{ kg}$$

$$F = 11,1 \text{ N}$$

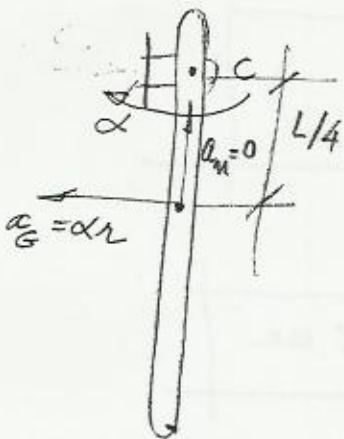
$$a) M_c = I_c \alpha$$

$$F_x \left(\frac{L}{2} + \frac{L}{4} \right) = \left(\frac{1}{12} m L^2 + m \left(\frac{L}{4} \right)^2 \right) \cdot \alpha$$

$$F \times \frac{3}{4} L = \frac{28}{192} m L^2 \alpha$$

$$11,1 \times \frac{3}{4} = \frac{28 \times 3,4}{192} \times 0,914 \alpha$$

$$\alpha = 18,37 \text{ rad/s}^2$$



$$\Sigma F_x = m a_x$$

$$F - C_x = m \cdot \alpha \bar{r}$$

$$11,1 - C_x = 3,4 \times 18,37 \times 0,229$$

$$C_x = -3,20 \text{ N}$$

$$C_x = 3,20 \text{ N} \rightarrow$$

$$\Sigma F_y = 0$$

$$-3,4g + C_y = 0$$

$$C_y = 3,4g$$

$$C_y = 33,35 \text{ N} \uparrow$$

16.81 - DO PROBLEMA ANTERIOR:

$$F - C_n = m \alpha \bar{r} \quad C_n = 0$$

$$F = m \alpha \bar{r} \quad \alpha = \frac{F}{m \bar{r}}$$

$$\Sigma M_o = I_o \alpha$$

$$F_x \left(\frac{L}{2} + \bar{r} \right) = m \left(\frac{L^2}{12} + \bar{r}^2 \right) \alpha$$

$$\cancel{F} \left(\frac{L}{2} + \bar{r} \right) = \cancel{m} \left(\frac{L^2}{12} + \bar{r}^2 \right) \cdot \frac{\cancel{F}}{\cancel{m} \bar{r}}$$

$$\frac{L}{2} \bar{r} + \bar{r}^2 = \frac{L^2}{12} + \bar{r}^2$$

$$\frac{\cancel{L}}{2} \bar{r} = \frac{L^2}{12 \cancel{L}}$$

$$\boxed{\bar{r} = \frac{L}{6}}$$

$$\alpha = \frac{F}{m \bar{r}}$$

$$\boxed{\alpha = \frac{6F}{mL}}$$

$F/$ $L = 0,914 \text{ m}$

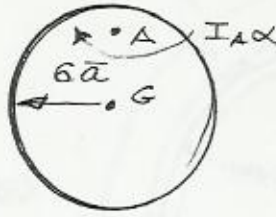
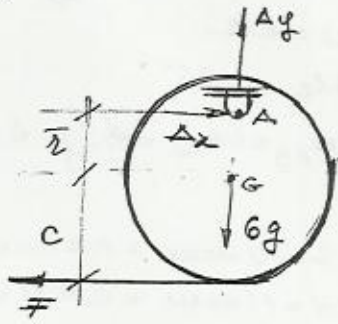
$m = 3,4 \text{ kg}$

$F = 11,1 \text{ N}$

$$\boxed{\bar{r} = 0,152 \text{ m}}$$

$$\boxed{\alpha = 21,43 \text{ rad/s}^2}$$

16.82-

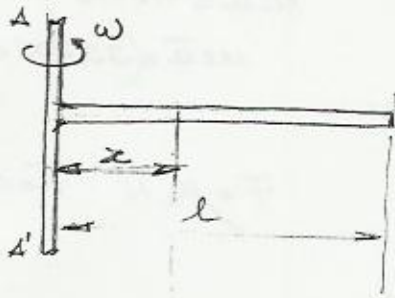


$C = 160 \text{ mm}$
 $m = 6 \text{ kg}$
 $F = 20 \text{ N}$
 $\bar{r} = \frac{3}{4} C = 120 \text{ mm}$

a) $M_A = I_A \alpha$
 $M_A = \left(\frac{m c^2}{2} + m \bar{r}^2 \right) \alpha$
 $F(\bar{r} + c) = m \left(\frac{c^2}{2} + \bar{r}^2 \right) \alpha$
 $\alpha = 34,31 \text{ rad/s}^2$

b) $\sum F_y = 0$
 $\Delta y = 6g \Rightarrow \Delta y = 58,86 \text{ N} \uparrow$
 $\sum F_x = m \bar{a}$
 $F - A_x = 6 \cdot (\alpha \times \bar{r})$
 $\Delta x = F - 6 \cdot (\alpha \times \bar{r})$
 $\Delta x = -4,70 \Rightarrow A_x = 4,70 \text{ N} \leftarrow$

16.85-

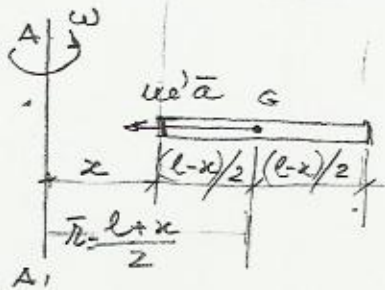
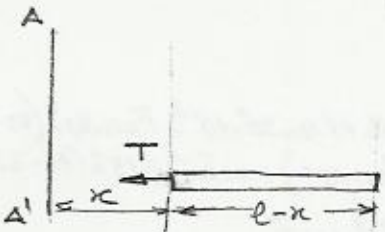


$\omega = \text{cte.}$
 $\frac{m'}{l-x} = \frac{m}{l} \Rightarrow m' = m \frac{l-x}{l}$
 $\bar{a} = a_m = \omega^2 \bar{r}$

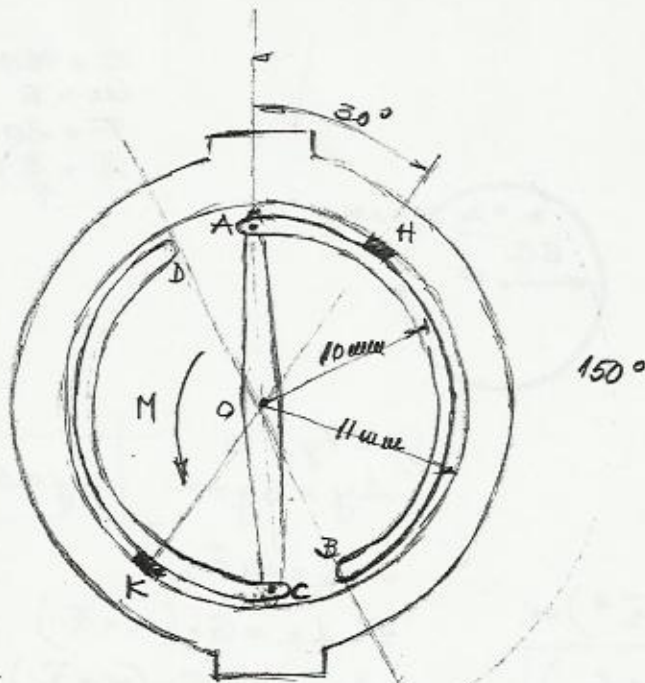
$\sum F_x = m' \bar{a}$
 $T = m' \bar{a}$
 $T = m \frac{l-x}{l} \cdot \omega^2 \frac{l+x}{2}$

$T = \frac{m \omega^2}{2} (l^2 - x^2)$

m



16.89-



$$\omega = 3000 \text{ rpm} = 3000 \frac{\text{r}}{30} = 100 \pi \text{ rad/s}$$

$$\omega = cte.$$

$$\mu_c = 0,35$$

$$\mu_{AB} = \mu_{CD} = 4,5g = 4,5 \times 10^{-3} \text{ kg}$$

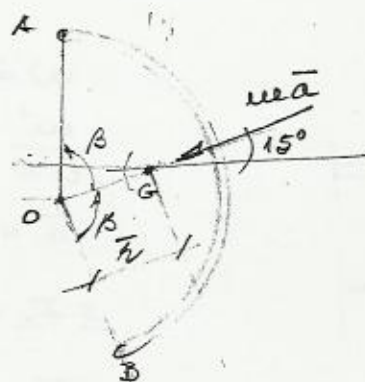
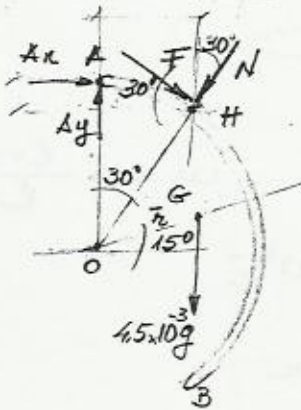
$$r = 10 \text{ mm} = 0,01 \text{ m}$$

$$OH = 11 \text{ mm} = 0,011 \text{ m}$$

$$2\beta = 150^\circ$$

$$\beta = 75^\circ \quad \beta = \frac{75\pi}{180} \text{ rad}$$

MEMBRO AB



$$\bar{r} = r \frac{\sin \beta}{\beta} \rightarrow \bar{r} = 0,0074 \text{ m}$$

$$\bar{r} = 7,38 \times 10^{-3} \text{ m}$$

$$\bar{a} = \omega^2 \bar{r}$$

$$\mu \bar{a} = 4,5 \times 10^{-3} \times \bar{r} \cdot \omega^2$$

$$\mu \bar{a} = (33,206 \times 10^{-6}) \omega^2$$

$$F = \mu_c N \quad F = 0,35 N$$

$$\Sigma M_A = (\Sigma M_A)_{cf. +}$$

$$4,5 \times 9,81 \times 10^{-3} \times 7,38 \times 10^{-3} \cos 15^\circ + N \sin 30^\circ \times (10 - 11 \cos 30^\circ) \times 10^{-3} + F \cos 30^\circ \times 11 \sin 30^\circ \times 10^{-3} - F \sin 30^\circ \times (10 - 11 \cos 30^\circ) \times 10^{-3} + N \cos 30^\circ \times 11 \sin 30^\circ \times 10^{-3} = \mu \bar{a} (\sin 15^\circ \times 7,38 \times 10^{-3} \cos 15^\circ) + \mu \bar{a} (\cos 15^\circ \times (10 - 7,38 \sin 15^\circ) \times 10^{-3})$$

$$5 N \times 10^{-3} + 2,3397 F \times 10^{-3} + 0,3258 \times 10^{-3} = 9,659 \times 10^{-3} \mu \bar{a}$$

$$F = 0,35 N \quad N = \frac{F}{0,35}$$

$$5 \times \frac{F}{0,35} + 2,3397 F + 0,3258 = 9,659 \mu \bar{a}$$

$$16,6254 F = 9,659 \mu \bar{a} - 0,3258$$

$$F = 0,58 \mu \bar{a} - 0,02$$

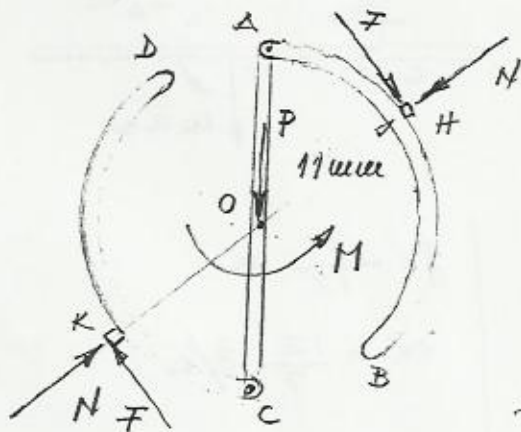
DESPREZANDO O 2º MEMBRO

$$F = 0,58 \mu \bar{a}$$

$$F = 19,292 \times 10^{-6} \omega^2$$

$$\mu \bar{a} = 33,206 \times 10^{-6} \cdot \omega^2$$

CONSIDERANDO AMBOS OS MEMBROS:



$$\alpha = 0 \quad \bar{a} = 0$$

$$\sum M_o = 0$$

$$M - 2F \times 11 \times 10^{-3} = 0$$

$$M = 2F \times 11 \times 10^{-3}$$

$$M = 2 \times 19,292 \times 11 \times 10^{-9} \omega^2$$

$$M = 424,42 \times 10^{-9} \omega^2$$

$$F / \omega = 100 \text{ N rad/s}$$

$$M = 0,042 \text{ N m}$$

$$M = 41,89 \times 10^{-3} \text{ N m}$$

$$M = 41,89 \text{ N mm}$$

16.90. DO PROBLEMA ANTERIOR:

$$M = 424,42 \times 10^{-9} \omega^2 \text{ N m}$$

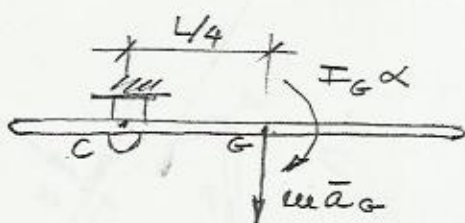
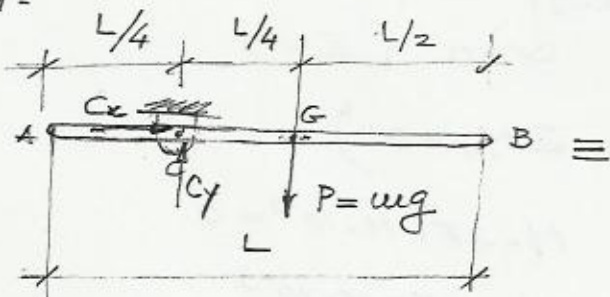
$$M = 50 \text{ N m} = 50 \times 10^{-3} \text{ N m}$$

$$\frac{50 \times 10^{-3}}{424,42 \times 10^{-9}} = \omega^2$$

$$\omega = 343,23 \text{ rad/s}$$

$$\omega = 3,278 \text{ rpm}$$

16.91-



$$\Sigma M_C = I_C \alpha$$

$$P \times L/4 = (I_G + m(\frac{L}{4})^2) \alpha$$

$$mg \times L/4 = \left(\frac{mL^2}{12} + \frac{mL^2}{16} \right) \alpha$$

$$g \times \frac{L}{4} = L^2 \left(\frac{1}{12} + \frac{1}{16} \right) \alpha$$

$$g = \frac{7L}{12} \alpha$$

$$\alpha = \frac{12}{7} g/L$$

a) $a_B = CB \times \alpha$

$$a_B = \left(L - \frac{L}{4} \right) \times \frac{12}{7} \frac{g}{L} = \frac{3L}{4} \times \frac{12}{7} \frac{g}{L}$$

$$a_B = \frac{9}{7} g \downarrow$$

b) $\Sigma F_x = m a_{Gx} = 0$

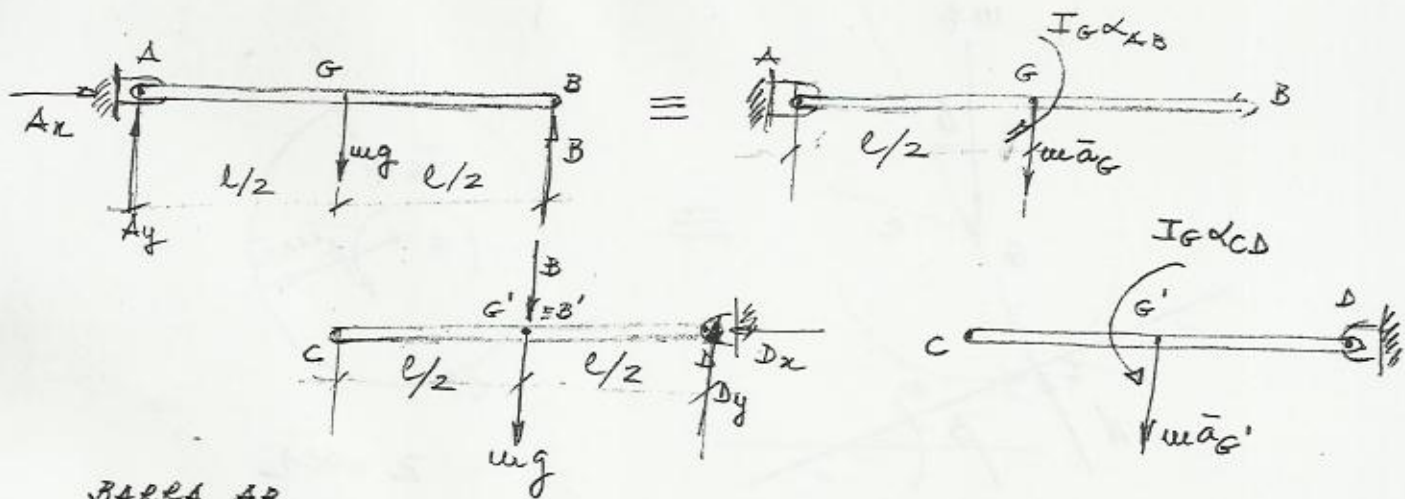
$$C_x = 0$$

$$\Sigma F_y = m a_{Gy} \Rightarrow P - C_y = m \bar{a}_G = m \alpha \times \frac{L}{4}$$

$$mg - C_y = m \frac{L}{4} \times \frac{12}{7} \frac{g}{L}$$

$$C_y = mg - \frac{3}{7} mg \quad C_y = \frac{4}{7} mg$$

$$C_y = \frac{4}{7} P \uparrow$$

BARRA AB

$$\omega_{AB} = 0$$

$$\Sigma M_A = I_A \alpha_{AB} \quad (+)$$

$$mg \frac{l}{2} - B l = \left[\frac{m l^2}{12} + m \left(\frac{l}{2} \right)^2 \right] \alpha_{AB}$$

$$B = \frac{mg}{2} - \frac{m l}{3} \alpha_{AB} \quad (1)$$

BARRA CD

$$\omega_{CD} = 0$$

$$\Sigma M_D = I_D \alpha_{CD} \quad (+)$$

$$mg \frac{l}{2} + B \frac{l}{2} = \left[\frac{m l^2}{12} + m \left(\frac{l}{2} \right)^2 \right] \alpha_{CD}$$

$$B = 2 \left(\frac{m l}{3} \alpha_{CD} - \frac{mg}{2} \right) \quad (2)$$

FAZENDO EQ (1) = EQ (2).

$$\frac{mg}{2} - \frac{m l}{3} \alpha_{AB} = 2 \left(\frac{m l}{3} \alpha_{CD} - \frac{mg}{2} \right)$$

$$\frac{g}{2} + g = \left(\frac{2}{3} \alpha_{CD} + \frac{1}{3} \alpha_{AB} \right) l$$

$$2 \alpha_{CD} + \alpha_{AB} = \frac{9}{2} \frac{g}{l} \quad (3)$$

CINEMÁTICA:

$$a_B = a_{B'}$$

$$\alpha_{AB} \times l = \alpha_{CD} \times \frac{l}{2} \Rightarrow \alpha_{AB} = \frac{\alpha_{CD}}{2} \quad (4)$$

$$a) a_C = \alpha_{CD} \times l \quad \boxed{a_C = 1,8g \downarrow}$$

b) SUBST. $\alpha_{CD} = 1,8g/l$ EM (2) TEMOS:

$$B = 2 \left(\frac{m l}{3} \times \frac{1,8g}{l} - \frac{mg}{2} \right)$$

$$B = \left(\frac{3,6}{3} - 1 \right) mg$$

$$\boxed{B = 0,2mg \uparrow}$$

SUBST. (4) EM (3):

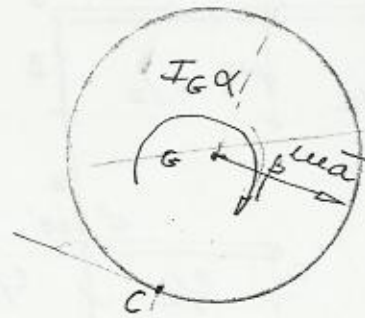
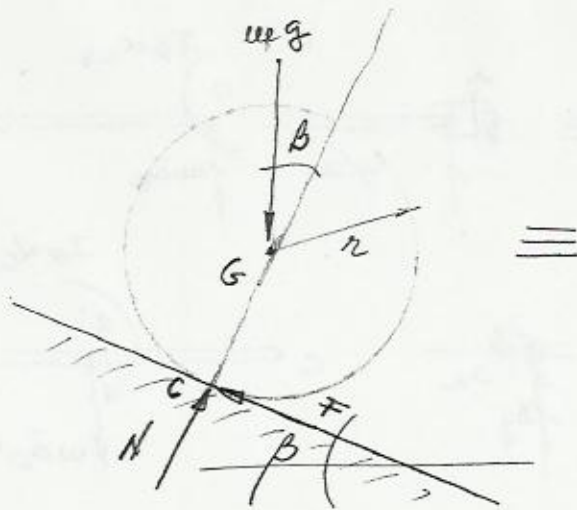
$$2 \alpha_{CD} + \frac{\alpha_{CD}}{2} = \frac{9}{2} \frac{g}{l}$$

$$\frac{5}{2} \alpha_{CD} = \frac{9}{2} \frac{g}{l}$$

$$\alpha_{CD} = \frac{9}{5} \frac{g}{l}$$

$$\alpha_{CD} = 1,8g/l \quad (5)$$

16.107-



$$\sum M_c = (\sum M_c)_{ef.}$$

$$\bar{a} = \alpha r$$

$$\sum M_c = I_c \alpha$$

$$mg \sin \beta \times r = I_G \alpha + m \bar{a} \times r$$

$$mg \sin \beta \times r = (I_G k^2 + m r^2) \alpha$$

$$\alpha = \frac{r \sin \beta g}{k^2 + r^2}$$

$$\bar{a} = \alpha r$$

$$\bar{a} = \frac{r^2}{k^2 + r^2} g \sin \beta$$

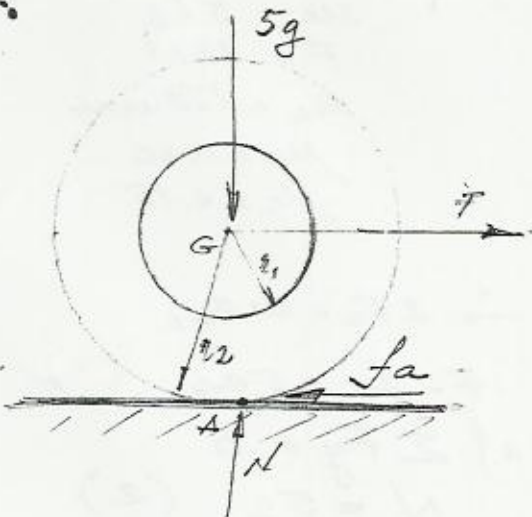


DIAGRAMA DO CORPO LIVRE

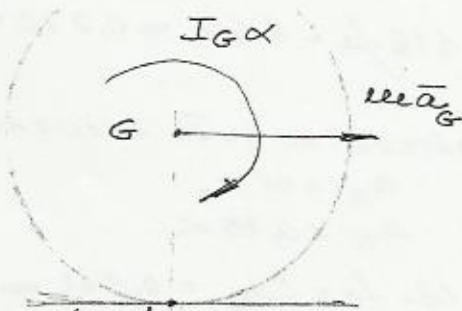


DIAGRAMA DAS FORÇAS RESULTANTES

$$r_1 = 80 \text{ mm}$$

$$r_2 = 160 \text{ mm}$$

$$m = 5 \text{ kg}$$

$$k_G = 120 \text{ mm}$$

$$F = 18 \text{ N}$$

$$\mu_e = 0,20$$

$$\mu_c = 0,15$$

$$\sum F_x = m a_{Gx}$$

$$F - f_a = 5 a_G \quad (1)$$

$$\sum F_y = 0$$

$$N = 5g \quad (2)$$

$$\sum M_G = I_G \alpha$$

$$f_a \times 0,16 = 5 \times 0,12^2 \alpha \quad (3)$$

1) ASSUMINDO A HIPÓTESE DE NÃO DESLIZAMENTO

$$a_G = \alpha r \quad a_G = 0,16 \alpha$$

$$18 - f_a = 5 \times 0,16 \alpha$$

$$0,16 f_a = 0,072 \alpha \Rightarrow \alpha = 2,22 f_a$$

$$18 - f_a = 0,8 \times 2,22 f_a$$

$$f_a = 6,48 \text{ N}$$

$$f_{a \max} = \mu_e N = 0,2 \times 5 \times 9,81 = 9,81 \text{ N}$$

$$f_a < f_{a \max}$$

O CORPO NÃO DESLIZA.

$$\alpha = 2,22 f_a \quad \alpha = 2,22 \times 6,48$$

$$\alpha = 14,4 \text{ rad/s}^2$$

$$a_G = 0,16 \alpha$$

$$a_G = 2,30 \text{ m/s}^2$$

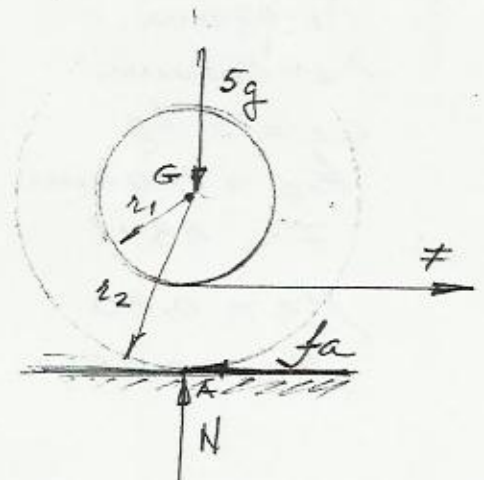


DIAGRAMA DO CORPO LIVRE

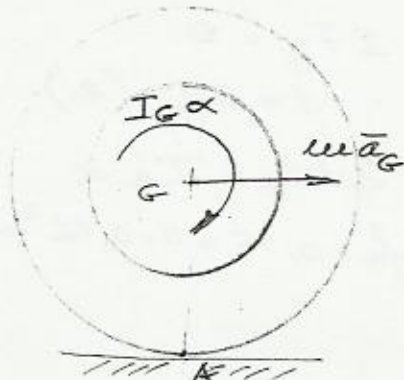


DIAGRAMA DAS FORÇAS RESULTANTES

- $r_1 = 80 \text{ mm}$
- $r_2 = 160 \text{ mm}$
- $m = 5 \text{ kg}$
- $F = 18 \text{ N}$
- $k_G = 120 \text{ mm}$
- $\mu_c = 0,20$
- $\mu_c = 0,15$

$$\rightarrow \sum F_x = m a_G$$

$$F - f_a = 5 a_G \quad (1)$$

$$\uparrow \sum F_y = 0$$

$$N = 5g \quad (2)$$

$$\curvearrowright \sum M_G = I_G \alpha$$

$$f_a \times r_2 - F \times r_1 = 5 \times 0,12^2 \alpha$$

$$0,16 f_a - 1,44 = 0,072 \alpha \quad (3)$$

1) HIPÓTESE DE NÃO DESLIZAMENTO

$$a_G = \alpha r$$

$$a_G = 0,16 \alpha$$

$$18 - f_a = 5 a_G = 0,8 \alpha \Rightarrow \alpha = \frac{18 - f_a}{0,8}$$

$$0,16 f_a - 1,44 = 0,072 \alpha$$

$$0,16 f_a - 1,44 = 0,072 \frac{18 - f_a}{0,8} \Rightarrow f_a = 12,24 \text{ N}$$

$$f_a = 12,24 \text{ N}$$

$$f_{a \text{ max}} = 0,20 \times 5 \times 9,81 \text{ N}$$

$$f_{a \text{ max}} = 9,81 \text{ N}$$

$$f_a > 9,81 \text{ N}$$

CONCLUSÃO: O CILINDRO ROLA DESLIZANDO.

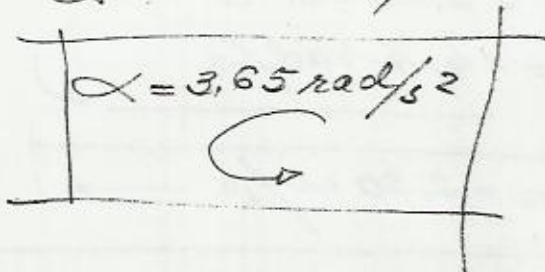
2). ASSUMINDO A CONDIÇÃO DE DESLIZAMENTO.

$$f_a = \mu_c N \quad f_a = 0,15 \times 5 \times 9,81 \text{ N} \quad f_a = 7,34 \text{ N}$$

$$0,16 f_a - 1,44 = 0,072 \alpha$$

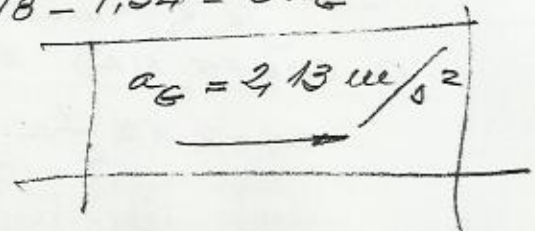
$$0,16 \times 7,34 - 1,44 = 0,072 \alpha$$

$$\alpha = -3,65 \text{ rad/s}^2$$



$$18 - f_a = 5 a_G$$

$$18 - 7,34 = 5 a_G$$



16.109 - 30 PROBLEMA 16.107, TEMOS:

$$\bar{a} = \frac{r^2}{r^2 + \bar{k}^2} g \operatorname{sen} \beta$$

$$\beta = 15^\circ$$

$$r = 25 \text{ mm}$$

$$\bar{k} = 400 \text{ mm}$$

$$\bar{a} = \frac{0,025^2}{0,025^2 + 0,4^2} 9,81 \times \operatorname{sen} 15^\circ$$

$$\bar{a} = 0,0099 \text{ m/s}^2 \quad \bar{a} = 9,88 \text{ mm/s}^2$$

$$\Delta \bar{x} = \frac{1}{2} \bar{a} t^2$$

$$T/t = 150 \quad \Delta \bar{x} = \frac{1}{2} \times 9,88 \times 15^2 \quad \Delta \bar{x} = 1.111,44 \text{ mm}$$

$$\Delta \bar{x} = 1,11 \text{ m}$$

16.130.

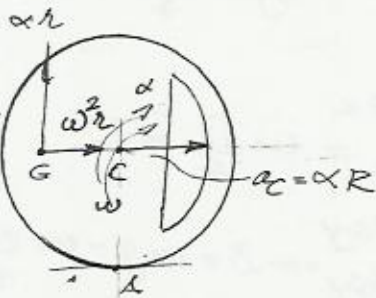
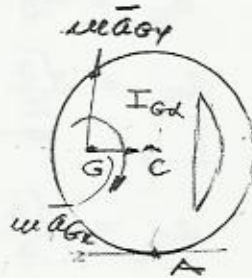
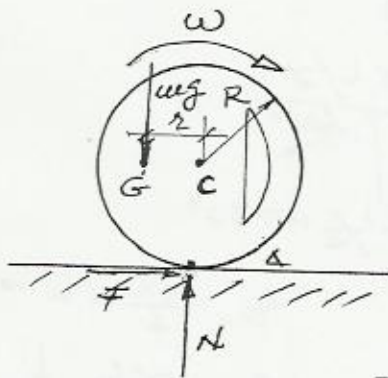
$$m = 5 \text{ kg}$$

$$R = 0,30 \text{ m}$$

$$r = 0,10 \text{ m}$$

$$\bar{k}_G = 0,15 \text{ m}$$

$$\omega = 8 \text{ rad/s}$$



$$\vec{a}_G = \vec{a}_C + \vec{a}_{G/C}$$

$$\vec{a}_G = \alpha R + \alpha r + \omega^2 r$$

$$a_{Gx} = \alpha R + \omega^2 r$$

$$a_{Gy} = \alpha r$$

$$\Sigma M_A = (\Sigma M_A)_{\text{efeb.}}$$

$$-mg \cdot r = I_G \alpha + m a_{Gx} R + m a_{Gy} \cdot r$$

$$-mg \cdot r = \frac{1}{2} m \bar{k}_G^2 \alpha + m (\alpha R + \omega^2 r) \cdot R + m \alpha r \cdot r$$

$$-g r = \alpha (\bar{k}_G^2 + R^2 + r^2) + \omega^2 r \cdot R$$

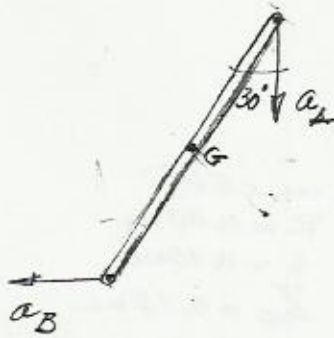
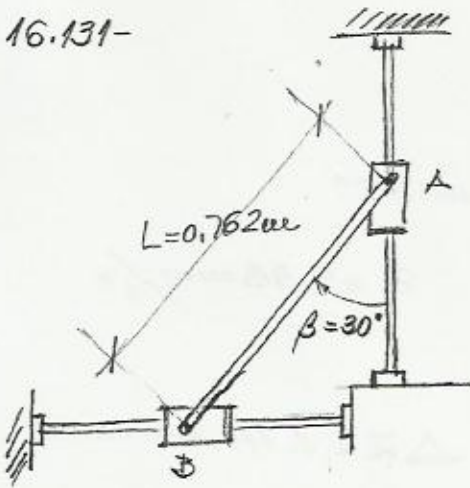
$$\frac{-(g + \omega^2 r) r}{\bar{k}_G^2 + R^2 + r^2} = \alpha$$

$$\alpha = -23,68 \text{ rad/s}^2$$

$$\alpha = 23,68 \text{ rad/s}^2$$

16.131-

$m = 2,72 \text{ kg}$
 $\omega_0 = 0$



$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$a_A = a_B + a_{A/B}$

$a_B = \alpha L \cos 30^\circ = \frac{\sqrt{3}}{2} \alpha L$

$\vec{a}_G = \vec{a}_B + \vec{a}_{G/B}$

$a_G = \frac{\sqrt{3}}{2} \alpha L + \alpha L/2$

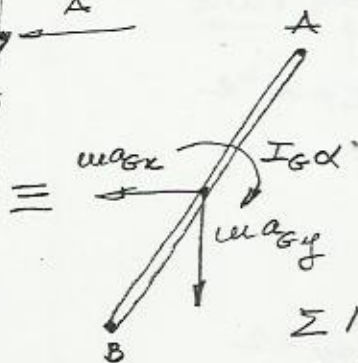
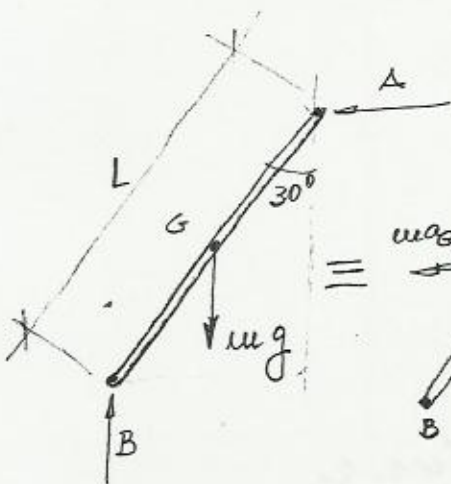
$a_{Gx} = -\frac{\sqrt{3}}{2} \alpha L + \alpha L/2 \cos 30^\circ$

$a_{Gx} = -\frac{\sqrt{3}}{4} \alpha L$

$a_{Gx} = \frac{\sqrt{3}}{4} \alpha L$

$a_{Gy} = -\alpha \frac{L}{2} \sin 30^\circ$

$a_{Gy} = \frac{\alpha L}{4}$



$\sum F_x = m a_{Gx}$

$A = m a_{Gx} = m \frac{\sqrt{3}}{4} \alpha L$

$\sum F_y = m a_{Gy}$

$mg - B = m a_{Gy} \Rightarrow B = mg - m \frac{\alpha L}{4}$

$\sum M_G = I_G \alpha$

$B \times \frac{L}{2} \sin 30^\circ - A \times \frac{L}{2} \cos 30^\circ = \frac{m L^2}{12} \alpha$

$(mg - \frac{m \alpha L}{4}) \times \frac{L}{2} \sin 30^\circ - \frac{m \alpha \sqrt{3} L}{4} \times \frac{L}{2} \cos 30^\circ = \frac{m L^2}{12} \alpha$

$g \frac{\sin 30^\circ}{2} = \alpha \left(\frac{L}{12} + \frac{L}{16} + \frac{3}{8} L \right)$

$\alpha = 6,18 \text{ rad/s}^2$

$A = m \frac{\sqrt{3}}{4} \alpha L$

$A = 5,55 \text{ N}$

$B = mg - m \frac{\alpha L}{4}$

$B = 23,48 \text{ N}$