AMERICAN JOURNAL of PHYSICS

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Citation: American Journal of Physics **82**, 349 (2014); doi: 10.1119/1.4862967
View online: http://dx.doi.org/10.1119/1.4862967
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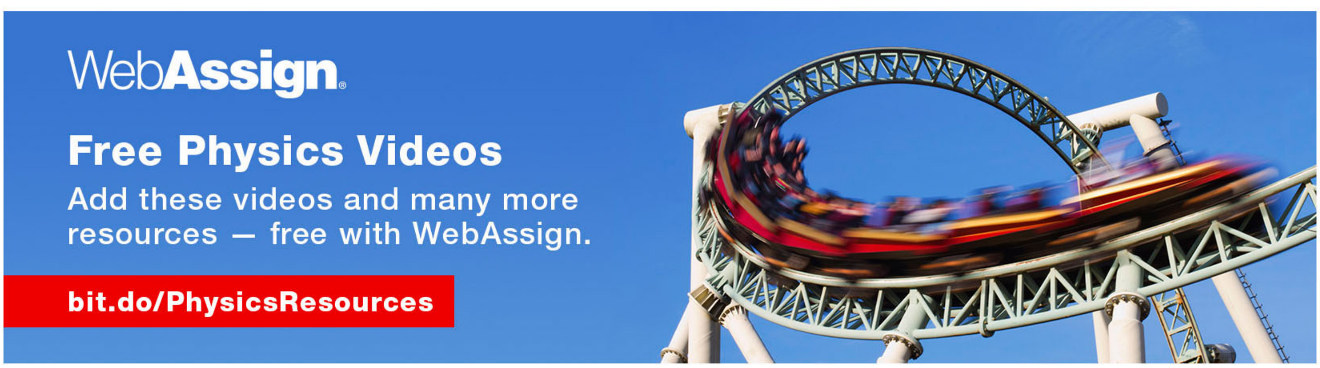

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Accurate physical laws can permit new standard units: The two laws $\vec{F} = m\vec{a}$ and the proportionality of weight to mass

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(Received 8 February 2013; accepted 10 January 2014)

Three common approaches to $\vec{F} = m\vec{a}$ are: (1) as an exactly true *definition* of force $\vec{F}$ in terms of measured inertial mass m and measured acceleration $\vec{a}$; (2) as an exactly true *axiom* relating measured values of $\vec{a}$, $\vec{F}$ and m; and (3) as an imperfect but accurately true *physical law* relating measured $\vec{a}$ to measured $\vec{F}$, with m an experimentally determined, matter-dependent constant, in the spirit of the resistance R in Ohm's law. In the third case, the natural units are those of $\vec{a}$ and $\vec{F}$, where $\vec{a}$ is normally specified using distance and time as standard units, and $\vec{F}$ from a spring scale as a standard unit; thus mass units are derived from force, distance, and time units such as newtons, meters, and seconds. The present work develops the third approach when one includes a second physical law (again, imperfect but accurate)—that balance-scale weight W is proportional to m—and the fact that balance-scale measurements of relative weight are more accurate than those of absolute force. When distance and time also are more accurately measurable than absolute force, this second physical law permits a shift to standards of mass, distance, and time units, such as kilograms, meters, and seconds, with the unit of force—the newton—a derived unit. However, were force and distance more accurately measurable than time (e.g., time measured with an hourglass), this second physical law would permit a shift to standards of force, mass, and distance units such as newtons, kilograms, and meters, with the unit of time—the second—a derived unit. Therefore, the choice of the most accurate standard units depends both on what is most accurately measurable and on the accuracy of physical law. *© 2014 American Association of Physics Teachers.*
[http://dx.doi.org/10.1119/1.4862967]

I. INTRODUCTION

The present work considers Newton's second law of motion as a *physical law* with a limited range of validity, as opposed to a *definition* or an *axiom* (both of which would be identically true). It then shows how, when one also uses the physical law that the weight W is proportional to the inertial mass m, the establishment of standards can be affected. Because it is important for our purposes to distinguish between definition and physical law, this introduction first distinguishes them, then discusses three viewpoints toward $\vec{F} = m\vec{a}$ (the two others being to treat it as a definition of $\vec{F}$ and to treat it as an axiom that yields $\vec{a}$) and then gives a brief commentary, as preparation for the body of the paper. Although the present work is pedagogical in nature, its viewpoint forms the philosophical basis for the shift from the standard units of meters, seconds, and newtons—the natural units of the second law—to meters, seconds, and kilograms—the natural units when these quantities are more accurately measurable than force.

A. The distinction between physical definition and physical law

By physical definition we mean a *named* relationship between measurable quantities. Thus we define velocity—that is, we give it a name—as $v \equiv dx/dt$, where dx is the change in distance x (measured, e.g., in meters) and dt is the change in time t (measured, e.g., in seconds), in the limit where $dt \to 0$. Applied to the speed of an automobile, we can make approximate measurements of dx and dt to obtain v directly from its definition. Because of uncertainties in the measurement of dx and dt, v cannot be known exactly, but this is not because there is any uncertainty in its definition.

By physical law, we mean a relationship among a number of independently measurable quantities. For example, consider a radar generator that produces electromagnetic radiation traveling with velocity c and of frequency f_0 in the rest frame of the generator. The physical law for the Doppler shift states that the frequency f of radar reflected from an object moving away from the generator with nonrelativistic velocity $v \ll c$ and measured in the generator frame is given by $f = f_0(1 - v/c)$. The experimentally established accuracy of a physical law depends on the accuracy of measurement of the quantities it relates; it is no more accurate than the least accurately measurable of these quantities.

Because it is easier to measure f_0, c, and f than to measure v by direct use of its definition, and because the physical law (for the Doppler shift) is highly accurate, this physical law provides a means to measure the velocity v of an automobile, via $v = c[1 - (f/f_0)]$, more accurately than by use of its direct definition. Therefore, it permits a more accurate standard for velocity, without a direct measurement (via its definition) of velocity.

In general, new physical laws of high accuracy, such as the ac Josephson effect and its associated voltage, or the quantum Hall effect and its associated electrical resistance, permit us to determine more accurate standards of measurement (in these cases, of voltage and of resistance).[1] In what follows we consider $\vec{F} = m\vec{a}$ from the viewpoint of both definition and physical law.

Typically, the use of accurate new physical laws to define new standards is a complex matter involving many quantities.[1] The present work considers a case that, although relatively simple, is nontrivial and can lead to multiple choices of standards, depending on which measurable quantities and which physical laws have the highest accuracy.

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B. Three approaches to $\vec{F}=m\vec{a}$

Given the distinction between definition and physical law, we now consider three views relative to the modern form of Newton's second law of motion,

$$\vec{F} = m\vec{a}. \tag{1}$$

(1) One can consider that the acceleration $\vec{a}$ and inertial mass m are measurable, but force $\vec{F}$ is not. From this perspective one assumes that $\vec{F} = m\vec{a}$ is exactly true and thus *defines* $\vec{F}$. We do not take this viewpoint.

Note that St. Venant[2] and Mach[3] assume momentum conservation to be an *exact* law, and use it to determine mass ratios of two interacting particles by taking ratios of either velocity changes or of accelerations. Their view of inertial mass is consistent with the above approach that defines $\vec{F}$ exactly.

(2) In his *Principia*,[4] Newton takes a rather complex viewpoint. Following a number of definitions (including one for mass, his "quantity of matter," and one for momentum, his "quantity of motion") he *axiomatizes* his laws of motion so $\vec{F} = m\vec{a}$ is taken to be exactly true.[5] However, his definition of mass (see following paragraph) was informed by his (dynamic) pendulum experiments, given in the *Principia*'s Book 3, Proposition 6, by which he concluded that balance-scale weight W is proportional to inertial mass m:

$$W \propto m. \tag{2}$$

Because this conclusion involved an analysis of the pendulum experiments using $\vec{F} = m\vec{a}$, Newton's definition of mass is not really a primary definition.

Newton defined m as the product of volume and density. This definition has been criticized[3,6] as circular because Newton does not define density. However, a number of authors note that in Newton's time density was equivalent to specific gravity, a measurable quantity.[4,7,8] For example, Newton's contemporary Boyle, whose law for gases involves the gravitationally based density, had a few years before the *Principia* devised methods to measure the specific gravity of solids and liquids both less dense and more dense than water.[9,10] Thus Newton took mass to be proportional to weight, which he had established (as an experimental law) by comparing the results of his pendulum experiments with the predictions of the second law.

With mass known, Newton considered that $\vec{F} = m\vec{a}$ gave the acceleration of an object of a given mass subject to a given force or sum of forces. Although from Galileo's work acceleration was well-defined operationally, Newton did not give an operational definition of force. As far as we can tell, perhaps because of the absence of force units in Newton's time, he largely worked with the force of gravity and he employed proportional reasoning, so this lack of an operational definition for force was not an impediment to his development of the *Principia*. Thus, Newton's second law as stated by Newton was the physical law that

$$\vec{a} \propto \vec{F}, \tag{3}$$

with proportionality constant inversely proportional to his definition of mass (itself based on an independently determined physical law). Note that both $\vec{a}$ and $\vec{F}$, while having different units, are vectors under rotation.

(3) An alternative view—and the view taken in the present work—is to consider $\vec{F} = m\vec{a}$ as a *physical law*. This view is clearest if the equation is put in the form

$$\vec{a} = \frac{\vec{F}}{m}. \tag{4}$$

Here the quantity m (the inertial mass) is a property of the object under study; thus, once the proportionality is established by measuring $\vec{F}$ (in newtons, say, using, e.g., a spring and Hooke's law) and $\vec{a}$ (in, say, m/s^2), the proportionality constant $1/m$ can be determined. This view is similar to that taken when establishing Ohm's law, where the current I is proportional to the voltage difference ΔV (for small enough ΔV that nonlinear effects do not appear), with $1/R$ being the proportionality constant: $I = \Delta V/R$. Just as determination of R once and for all permits one to relate other values of voltage difference and current, so determination of m once and for all permits one to relate other values of force and acceleration.

Having established $\vec{F} = m\vec{a}$, one can proceed to make further and independent discoveries. From spring-based weight measurements for various objects one finds the physical law that an object's weight W is proportional to its inertial mass m. Moreover, from projectile motion under terrestrial gravity, or from pendulum experiments *à la* Newton, one finds that the local gravitational field strength g is the same for all objects. This is consistent with the weight measurements when one uses $a = F/m$ and employs g for a and W for F. The non-obvious fact that weight and inertial mass are proportional is not contained in Newton's second law; neither is the non-obvious fact that electric charge and inertial mass are not proportional. It is the primary purpose of this work to show that if the proportionality of weight to mass can be determined accurately enough, then it is possible to obtain new standards, replacing the least accurately known of force, distance, and time by the measurement of (relative) mass by a scale balance. Currently, force is the least accurately measurable of these quantities, but we point out that if distance or time were the least accurately measurable, then the choice of standards would be different.

C. Commentary

Most textbooks assume that $\vec{F} = m\vec{a}$ is an exact relationship, perhaps with the caveat that it is limited by its neglect of special relativity, general relativity, and quantum mechanics.[11] They likewise assume the exact proportionality of W to m. This assumption of exactness has perhaps made it easier for some authors to go further, and to accept viewpoint #1 above, that $\vec{F} = m\vec{a}$ can be used to *define* force $\vec{F}$ if m and $\vec{a}$ can be measured accurately. Nevertheless, from the viewpoint of physical law and standards, the accuracy of $\vec{F} = m\vec{a}$ first must be determined (which requires independent measurements of $\vec{F}$, m, and $\vec{a}$), after which $\vec{F} = m\vec{a}$ may (or may not) be employed to define a new standard for force. Currently, if mass is determined by proportionality to weight using a scale balance, then force is indeed the least accurately known of the quantities appearing in $\vec{F} = m\vec{a}$, so the proportionality of mass to weight can be employed to obtain a *standard* for $\vec{F}$ from accurate measurements of m and $\vec{a}$.

This choice of the "best" set of standards, however, is a bit arbitrary. To see this, imagine a civilization with an hourglass chronometer. In this case, $\vec{a}$ likely would be the least

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accurately known of the quantities appearing in $\vec{F} = m\vec{a}$, so this law could be employed to obtain a standard for $\vec{a}$, and then for time t. Similar considerations apply if distance is the least accurately known quantity appearing in $\vec{F} = m\vec{a}$. Clearly, the use of Newton's $\vec{F} = m\vec{a}$ depends upon what is known and how accurately it is known. (Some other works that take the view that $\vec{F} = m\vec{a}$ should be established experimentally, rather than postulated, are given by Refs. 12–14.)

An outline of the remainder of this work is as follows. Section II considers how one might establish $\vec{F} = m\vec{a}$ is a physical law, and Sec. III considers how one might establish the physical law that weight and mass are proportional. Section IV shows how one can use these laws, if accurate enough compared to the primary measurements of force, length, and time, to replace the least accurately known of these primary measurements. Finally, Sec. V provides a brief summary.

II. ESTABLISHING NEWTON'S LAWS OF MOTION IN A WORLD WITH A DIFFERENT HISTORY

We first note that Newton's first and third laws involve statics, whereas the second law does not.

The third law, of action and reaction, is a statement about static forces in some reference frame, and assumes that even when there is motion relative to that reference frame, the law holds (although the values of the forces may change). To establish this law quantitatively requires an absolute force-measuring device, such as a spring scale (or even a digital weight scale, which is force-based), as opposed to a balance scale, such as often found in a doctor's office, which can only measure relative weights. In principle, one can establish that the third law holds even in non-inertial reference frames.

The first law, stating that for unforced motion an object moves at a constant velocity, is not on its face a law about statics. However, since it merely involves a system that moves at a constant velocity relative to an inertial frame, it is applicable to the statics of forces in the moving reference frame. It states that, in an inertial reference frame (one that is at rest or moving at a constant velocity relative to the "fixed" stars), if there is no net force then the velocity is constant. This is often associated with the concept of inertia. However, note that if we are at rest relative to an inertial laboratory frame and we then walk at a constant velocity $\vec{v}$, then all other objects develop a velocity $-\vec{v}$, independent of their inertial mass. Thus, from the viewpoint of kinematics at constant velocity, the inertial mass is irrelevant to the first law. To establish the first law requires a true force-measuring device, so that one can show that the net force is zero, and devices for measuring distance and time, so that one can show that the velocity $d\vec{r}/dt$ is constant. Thus, even without the second law of motion there is a need for distance, force, and time units.

With this in mind, let us consider how one might establish Newton's laws of motion, beginning with the statics-associated first and third laws.

A. Spring scales measure force and establish the third law

Let there be a group of scientists who measure forces $\vec{F}$ with spring scales (based on Hooke's law) calibrated by the local weights of standard masses, where a balance scale is used to compare weights. Once the spring scales are calibrated, they can be used to measure weights in another laboratory. Now consider the interaction of objects 1 and 2. With $\vec{F}_{12}$ the force on object 1 due to object 2, the scientists find, to experimental accuracy yet to be specified, that $\vec{F}_{21} = -\vec{F}_{12}$, which is the third law—action and reaction. Note that Hooke's law is not needed to measure a force; a nonlinear but reproducible, and therefore non-hysteretic, spring would also work. Let us assume that by this means they can measure $\vec{F}$ in newtons to a part in 10^3; thus they can establish the third law to a part in 10^3.

I find it curious that Newton did not indicate how to measure force directly. Hooke's law dates to around 1660,[15] some thirty years before Newton's *Principia*, which refers to some of Hooke's astronomical measurements but not to Hooke's law. Since Newton and Hooke were contemporaries and occasionally corresponded (with no love lost between them[16]), Newton presumably knew Hooke's law. Consistent with Newton's *Principia*, Euler's influential works *Mechanica* (1736), on point mechanics, and *Theoria Motus Corporum Solidorum seu Rigidorum* (1765), on rigid bodies, do not seem to invoke Hooke's law or any other method to measure forces,[17] and only briefly discuss distance and time measurements. (It is perhaps of interest to note that the first commercial spring balance may have been made in England around 1760,[18] long after Hooke's discovery.) In a cursory examination of the physics literature on $\vec{F} = m\vec{a}$, the earliest reference I have found that treats force as a primary quantity is Maxwell's 1876 proposal to measure force with elastic threads.[19]

B. Kinematics measurements (rulers and clocks) measure velocity and acceleration

Let there be another group of scientists who measure space ($\vec{r}$) and time (t) using rulers and clocks, so they also can determine velocity ($\vec{v} \equiv d\vec{r}/dt$) and acceleration ($\vec{a} \equiv d\vec{v}/dt$). Let us assume that they can measure distances to a part in 10^4 and times to a part in 10^5. Then velocities (and accelerations) can be measured to a part in 10^4 (determined by the least accurate of the units associated with the acceleration, which is distance). At this point, we do not consider the nature of the forces that cause the acceleration.

C. Spring scales and kinematics measurements (rulers and clocks) together establish the first law

Suppose that, in an inertial frame, measurements with rulers and clocks yield that the acceleration $\vec{a}$ of an object is zero. Simultaneous measurements using spring scales yield that the total force $\vec{F}$ on the object is zero. Thus, together the two approaches have established the first law. Because $\vec{a}$ is measured to a part in 10^4, and $\vec{F}$ to a part in 10^3, the first law is established to a part in 10^3. Note that if we plot $|\vec{a}|$ vs. $|\vec{F}|$ the first law gives only the intercept of the plot; this does not forbid a dependence on $|\vec{F}|$ that is more complex than linear (e.g., quadratic, cubic, or linear plus quadratic).

D. Spring scales and kinematics measurements (rulers and clocks) together establish the second law and inertial mass

Like the first law, the second law involves measurements of force (using spring scales) and of acceleration (using

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rulers and clocks). For a nonzero net force $\vec{F}$ one finds that the acceleration $\vec{a}$ is along $\vec{F}$, and that $|\vec{a}|$ is linearly proportional to $|\vec{F}|$. This proportionality is basically the second law of motion, and the proportionality constant we call $1/m$.

Because $|\vec{a}|$ is measured to a part in 10^4 but $|\vec{F}|$ is measured to a part in 10^3, we can determine the inertial mass m to a part in 10^3 via the measured slope of a plot of $|\vec{a}|$ vs. $|\vec{F}|$.

III. A NEW PHYSICAL LAW: PROPORTIONALITY OF WEIGHT AND INERTIAL MASS

In what follows we solely consider terrestrial gravity. We assume that the law of universal gravitation has not yet been discovered.

A. Kinematics measurements (rulers and clocks) establish the law of fall

Our hypothetical scientists can measure velocities and accelerations (no matter the cause of the acceleration) to a part in 10^4. They now consider accelerations solely caused by terrestrial gravity. Such use of rulers and clocks yields, when air resistance can be neglected (as in Boyle's experiments using an evacuated chamber), that to a part in 10^4 all objects fall under gravity with the same local acceleration $|\vec{a}| = g$ (which can depend on locale). Similar conclusions can be made for pendulum motion using clocks alone, if the mass distributions and pendulum lengths are the same. If Newton's second law is assumed to be correct, and if the force of gravity is taken to be the weight W, then we conclude that $g = W/m$ is the same for all objects: the weight is proportional to the mass.

B. The balance scale and a standard of mass: Terrestrial measurements

Our scientists can also use a set of objects of known mass, and measure their relative weights using a balance scale (rather than a spring scale). They find that, to a part in 10^3, W (from statics) and m (from dynamics, using a force other than gravity) are proportional, independent of the specific values of W and m, and the proportionality is independent of the material studied, although the ratio W/m may vary from locale to locale.

C. Accurate relative weight measurements

We now assume that our scientists also find that their relative weight measurements are more reproducible than the spring scale measurements; say, to a part in 10^6. Nevertheless, to use these measurements to advantage, they need only assume that the ratio of weight to mass is accurate only to a part in 10^4, the same as distance (the less accurate of distance and time).

IV. REDEFINING STANDARDS

A. Redefining the standard of force using both Newton's second law and the proportionality of weight and inertial mass

At this point another group of scientists comes along and suggests that perhaps Newton's second law is more accurately true than previously established, to some as yet unknown accuracy. They then try to invert the logic and use: (1) the accuracy of $\vec{F} = m\vec{a}$ (assumed accurate to a part in 10^4), (2) the accuracy of scale balance measurements of masses (assumed accurate to a part in 10^4), and (3) the accuracy of ruler and clock measurements of accelerations (accurate to a part in 10^4). In this way they *obtain a new standard*, accurate to a part in 10^4, for force $\vec{F}$ (by direct measurement accurate only to a part in 10^3). Nevertheless, this does *not define* $\vec{F}$, which continues to come from a true force-measuring device.

Because acceleration was determined in the context of the second law to a part in 10^4, acceleration is necessarily accurate for the first law to a part in 10^4. However, the third law is independent of the others and must be tested to a part in 10^4. If the third law is found to be true to at least that accuracy—which we assume to be the case—then the assumptions that the second law is accurate to a part in 10^4, and that the proportionality of weight to mass is accurate to a part in 10^4, are both consistent. This consistency would thus establish the third law to a much higher degree of accuracy (a part in 10^4) than originally established (to a part in 10^3)—a significant advancement.

To summarize, we have a consistent scheme whereby the accuracy of measurement of force is increased by using two very accurate physical laws ($\vec{F} = m\vec{a}$ and $W \propto m$), and then switching from the original distance-force-time-based standard units to the more accurate distance-mass-time-based standard units. Nevertheless, we see that $\vec{F} = m\vec{a}$ does not define $\vec{F}$; it rather serves to define a more accurate standard for $\vec{F}$.

B. Redefining the standards of time and distance using Newton's (accurate) second law

If time were the least accurately measurable quantity of distance, force, and time (e.g., time is measured with an hourglass), then in the scalarized version of the second law, $F = ma$, the acceleration a would be the least accurate measurement. In this case we may use $a = F/m$ to obtain a new standard for a. Because the rulers yield accurate enough values of distance, this approach would lead to a new standard of time.

If distance were the least accurately measurable quantity (e.g., measured with a grade-school ruler), then, in the scalarized version of the second law, $F = ma$, the acceleration a again would be the least accurate measurement. Again we may use $a = F/m$ to obtain a new standard for a. Because the clocks yield accurate enough values of time, this approach would lead to a new standard of distance.

We thus see how an accurate Newton's second law can be used to obtain either a new standard of time or of distance, without serving to define either time or distance.

V. SUMMARY AND CONCLUSION

We have shown how, by treating $\vec{F} = m\vec{a}$ as a physical law and by using the physical law that the weight W is proportional to the inertial mass m, one can, with accurate balance scale measurements of relative weight, obtain a new standard of force, distance, or time, according to which is the least accurately measurable of the three. Presently, force is the least accurately measurable of these quantities, leading to the use of distance-mass-time units rather than distance-force-time units.

These possibilities occur when the measurements appropriate to a physical law, and the physical law itself, are more

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accurately true than the measurements needed to obtain the definition of a physical quantity. We can then obtain that physical quantity more accurately by application of the physical law than by application of the definition. In other words, the choice of the most accurate standard units depends both on what is most accurately measurable and on the accuracy of physical law.

In a more modern context, the ac Josephson effect defines a voltage standard, not voltage itself, and the quantum Hall effect defines an electrical resistance standard, not electrical resistance itself. The efforts at laboratories like the National Institute for Standards and Technology are not merely practical, but are also epistemological, treating the very foundations of what we know and how we know it.

ACKNOWLEDGMENTS

The author would like to thank E. Tal and P. Mohr for communications relating to standards, I. Bruce and M. Nauenberg for providing historical references, M. Cohen for general discussions, and G. Entine and K. Krisciunas for comments on the manuscript. The author would also like to thank an anonymous referee who provided valuable historical perspective and references.

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[1]P. J. Mohr and D. B. Newell, "The physics of fundamental constants," Am. J. Phys. **78**, 338–358 (2010).

[2]A. J. C. B. St. Venant, *Principes de Méchanique fondés sur la Cinématique* (Bachelier, Paris, 1851), Section 81.

[3]E. Mach, *Science of Mechanics* (The Open Court Publishing Company, Chicago, 1902). On p. 241 he remarks that "The concept of mass is not made clearer by describing mass as the product of the volume into the density, as density itself denotes simply the mass of unit of volume. The true definition of mass can be deduced only from the dynamical relations of bodies."

[4]Isaac Newton, *The Principia*, translated from the Latin by I. B. Cohen and Anne Whitman (University of California Press, Berkeley, 1999). The original Principia was published in 1687, followed by revisions in 1713 and 1726. Quotations from the Principia are taken from this translation.

[5]Newton did not have units for force or mass ("quantity of matter"), or even for velocity or acceleration, and considered what we would call force, pressure, and impulse as all capable of producing motion according to the second law. His second law was, literally taken, the statement that change in momentum ("quantity of motion") is proportional to impulse. Apparently, the first time that the form $F = ma$ appears is in Jacob Hermann, *Phoronomia* (Rod. and Gerh. Wetstenios, Amsterdam, 1716). See p. 56 of Book I, *On the Forces and Motions of Bodies*, where the form $G = MV/T$ appears for a constant force G, mass M, velocity V, and time T. This work currently is available online at <http://archive.org/details/phoronomiasived00conggoog>.

[6]W. Thomson and P. G. Tait, *Treatise on Natural Philosophy* (Cambridge U. P., Cambridge, 1879). On p. 220 of vol. 1, part 1, they remark that "in reality the definition gives us the meaning of density rather than of mass."

[7]M. Jammer, *Concepts of Mass in Classical and Modern Physics* (Harper, New York, 1961).

[8]George Smith, "Newton's *Philosophiae Naturalis Principia Mathematica*," in *The Stanford Encyclopedia of Philosophy* (Winter 2008 edition), edited by Edward N. Zalta; available online at <http://plato.stanford.edu/archives/win2008/entries/newton-principia/>.

[9]R. Boyle, "An Account of a New kind of Baroscope, which may be called Statical; and of some Advantages and Conveniences it hath above the Mercurial," Philos. Trans. R. Soc. London **14**, 231–239 (1666), available online at <https://archive.org/details/philtrans07839158>.

[10]R. Boyle, "A New Essay-Instrument invented and described by the Honourable Robert Boyle, together with its uses thereof," Philos. Trans. R. Soc. London **115**, 329–348 (1675), available online at <https://archive.org/details/philtrans05533774>.

[11]For a discussion emphasizing the limitations imposed by relativity on the definition of mass, see E. Hecht, "There is no really good definition of mass," Phys. Teach. **44**, 40–45 (2006).

[12]F. W. Sears, *Mechanics, Wave Motion, Heat* (Addison-Wesley, Reading, MA, 1958). This text begins with vector composition (especially force) in Chapter 1, establishing the first and third laws and doing statics in Chapter 2, introducing moment and doing more statics in Chapter 3, covering one-dimensional kinematics in Chapter 4, and then experimentally establishing the second law in Chapter 5. Although different units are discussed, the accuracy of each of these laws is not considered.

[13]A. P. French, *Newtonian Mechanics* (W. W. Norton, New York, 1971), p. 166.

[14]M. Cohen, *Classical Mechanics: A Critical Introduction* (unpublished, 2011), available online at <https://www.physics.upenn.edu/resources/online-textbook-mechanics>.

[15]R. Hooke, *Lectures de potentia restitutiva, or, Of spring, explaining the power of springing bodies* (J. Martyn, London, 1678). The beginning of this work explains that in 1660 Hooke discovered the law of elasticity which bears his name and which describes the linear variation of tension with extension in an elastic spring, and which he first described in the anagram "ceiiinosssttuv," whose solution he reveals in this book as "Ut tensio, sic vis," meaning "As the extension, so the force." Hooke discusses both springs and coils, noting that he held off on stating his law in 1660 because he was in the process of acquiring a patent on a watch using his law. Hooke also notes that in his Micrographia he discusses the elastic properties of gases.

[16]J. Gleick, *Isaac Newton* (Vintage Books, New York, 2003).

[17]See *Euler*: *Some Papers*, translated and edited by Ian Bruce *et al*., available online at <www.17centurymaths.com/contents/eulercontents.html>. This is an extraordinary resource for various Euler texts that have been translated from the Latin. Some of them, associated with logarithms, have been the basis for I. Bruce, "Napier's Logarithms," Am. J. Phys. **68**, 148–154 (2000).

[18]Wikipedia, "Spring scale," <http://en.wikipedia.org/wiki/Spring_scale>, attributed there to Christian H. Hewison, *Locomotive Boiler Explosions* (David and Charles, Devon, England, 1983), p. 18.

[19]J. C. Maxwell, *Motion and Matter* (Van Nostrand, New York, 1876). This work discusses mass as derived from force and the second law on pp. 62–69.

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